Multi-channel DFE equalization with waveguide constraints for underwater acoustic communication

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***Abstract*—Although the multichannel decision feedback equalizer (DFE) has been shown to be nearly optimal and very effective for handling the difficulties of the underwater communications channel, this technique has been slow to be implemented in operational systems due to its high computational complexity. In this paper, we propose using measurements commonly available to oceanographic systems, such as depth, range, and speed of sound, to create a model of the arrival structure at a receiver with multiple elements. Three structures are presented which take advantage of this model to constrain the complexity of the multichannel DFE: a beamspace approach, a time-aligned beamspace approach, and an approach which uses discrete prolate spheroidal functions. Each of these approaches is integrated into a multichannel, direct adaptation DFE which is implemented using a recursive least squares (RLS) algorithm. The proposed structures are tested using the SPACE08 data set across a range of environmental conditions and using several exponential forgetting factors. It is found that these constrained approaches provide significant computational advantages over the full sensor-space approach and performance advantages over other computationally similar algorithms.**

I. Introduction

Currently, there is a disconnect between methods proposed in the literature for improving underwater acoustic communication and what is implemented in operational systems. A major reason for this disconnect is that the methods proposed in the academic literature tend to have high computational complexity and are thus impractical for the types of hardware currently available for use.

Multichannel equalization was shown in [1] to be a nearly optimal method for handling the difficulties of the underwater communication channel. Operational systems have been slow to adopt this method die to its computational complexity. In this paper, we present a method which uses measurements typically available to oceanographic systems, such as depth, range, and speed of sound, to create a model of the arrival structure of energy at a receiver. This model will be used to constrain the look directions of a beamformer and thus to reduce the complexity of the multichannel equalizer.

In their seminal work on adaptive equalization for underwater communication, Stojanovic et al [1] found that the optimal multichannel combiner, assuming the channel is known, is the sampled sum of the individual channel matched filters followed by a maximum likelihood sequence estimator (MLSE). Rather than implement this approach in its full complexity, the authors recommend using a decision feedback equalizer (DFE) in place of the MLSE and furthermore show that empirically, and adaptive multichannel DFE is nearly optimal in the underwater channel.

In a subsequent paper [2], the same set of authors demonstrated that introducing a beamformer into the framework was mathematically equivalent to using the full sensor space, when the direction of arrival of the received signals is known. When the directions are not known, the authors showed as long as the observed beamspace spans the received signal space and the observation noise is spatially white, there is no loss of performance from using a beamformer to pre-process the data.

Stojanovic also noted that, unlike many array processing application, underwater communication systems did not seek to eliminate multipath arrivals as interference, but instead sought to collect all the signal energy at the receiver. This changed the strategy for array receivers from using the beamformer to separate arrivals from each other to using the beamformer to gather and coherently combine the energy from all arrival paths.

In this work, we build on this idea of energy collection. The difference in our approach comes from the observation that while we often don't know the exact angles of arrival, we can estimate roughly the angular spread of the incoming arrivals based on the transmitter / receiver geometry. From a model of the arrival structure, we propose creating an orthogonal set of beams which span the estimated range of arrival angles to collect the incoming signal energy. The resulting beams are fed into a multichannel direct-adaptation DFE and the received symbols are estimated.

The idea of using physical constraints to improve underwater acoustic signal processing is not a new one. Kraay and Baggeroer [3] proposed the idea of a physically constrained method for array processing. They proposed using narrowband plane waves across the array to constrain the estimated signal covariance matrix to be one that could be realized by signals obeying the propagation constraints. Their goal was to reduce the number of snapshots needed to properly estimate a covariance matrix.

Papp et al [4], [5] applied this idea to mode processing in the application of underwater acoustic communications. They showed that it is possible to use mode filtering to improve array processing. However, when applied to equalization, a direct-adaptation equalizer with no pre-processing had lower residual mean-squared-error than the mode filtering approach. Furthermore, these approaches focused on reducing the amount of data needed for estimation without focusing on the computational complexity. As a result, these approaches require high computational complexity.

LeBlanc and Beaujean [6] proposed applying principle component analysis (PCA) to acoustic communication systems with receive arrays to improve equalizer performance. The received data correlation matrix is decomposed into its eigenvalue - eigenvector form to determine the signal-subspace. This method appears to hold promise in terms of reducing mean squared error, but requires a form of subspace tracking, which has high computational complexity.

Our proposed method of using an arrival structure model reduces the complexity of the resulting multichannel equalizer without introducing much additional overhead complexity. If the transmitter and receiver are both stationary, the model can be computed ahead of time. Even when the transmitter or receiver (or both) are moving, the change in the observed physical parameters is very slow compared with the data rate.

The remainder of this paper is organized as follows: section II describes multichannel decision feedback equalization, Section III describes beamforming, and Section IV describes the acoustic propagation model for shallow water. Section V includes the description of three proposed equalizer implementations. Section VI shows our validation of these ideas using experimental evidence and section VII contains conclusions and final thoughts.

Throughout this paper, lowercase bold letters, $\mathbf{a}$, indicate vectors, uppercase bold letters, $\mathbf{A}$, indicate matrices, and non-bold symbols are used for scalars. The symbols ^{T} indicates the transpose of a quantity, * the conjugate, and ^{H} the conjugate transpose or Hermitian. All vectors are assumed to be column vectors.

II. Decision feedback equalization

The decision feedback equalizer (DFE) consists of two linear filters working together: the feedforward filter collects the energy from the received signal and shapes its response and the feedback filter cancels the inter-symbol interference (ISI) from previously received symbols [7]. The general DFE equation can be written as:

$$\tilde{d}[n] = \sum_{\ell=-L_a}^{L_c-1} a^*[\ell]u[n-\ell] + \sum_{\ell=1}^{L_{\text{fb}}} b^*[\ell]\hat{d}[n] \qquad (1)$$

where $u[n]$ is the baseband received data, $\hat{d}[n]$ is the past symbol decisions, and $\tilde{d}[n]$ is the filtered received data before a symbol decision has been made. The feedforward filter coefficients are represented as $a[n]$ and the feedback coefficients as $b[n]$. The total number of DFE coefficients is $L = L_a + L_c + L_{\text{fb}}$.

The DFE equation can be represented more compactly using vector notation as:

$$\tilde{d}[n] = \mathbf{a}^H\mathbf{u}[n] + \mathbf{b}^H\hat{\mathbf{d}}[n-\ell] = \mathbf{h}^H\mathbf{z}[n] \qquad (2)$$

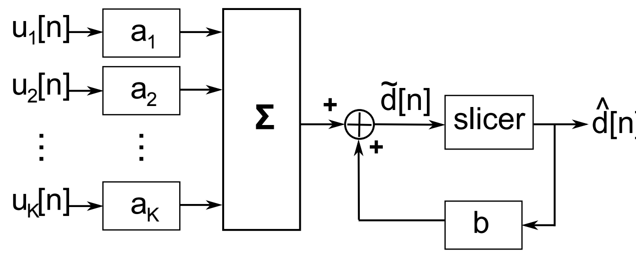

Fig. 1. A multichannel decision feedback equalizer.

where $\mathbf{h}^T = [\mathbf{a}^T \ \mathbf{b}^T]$ is a vector of filter coefficients and $\mathbf{z}^T[n] = [\mathbf{u}^T[n] \ \hat{\mathbf{d}}^T[n]]$ is a data vector containing both the received data and the past symbol estimates.

This framework can be modified to accommodate multiple receivers by expanding the definition of the filter and data vectors to be:

$$\mathbf{h} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_K \\ \mathbf{b} \end{bmatrix} \qquad \mathbf{z}[n] = \begin{bmatrix} \mathbf{u}_1[n] \\ \mathbf{u}_2[n] \\ \vdots \\ \mathbf{u}_K[n] \\ \hat{\mathbf{d}}[n] \end{bmatrix}$$

where there are K receive elements, $\mathbf{u}_i[n]$ is the vector of data received at the $\mathrm{i^{th}}$receive element, and $\mathbf{a}_i$ is the vector of feedforward filter coefficients for the $\mathrm{i^{th}}$receive element. See Figure 1 for an illustration of the functionality of a multichannel DFE.

It is common to use a fractionally sampled equalizer for timing [8]. Using the supplied framework, the feedforward filters will each have r_{fs} samples per received symbol, while the feedback filter will remain the same length. At each iteration, the data fed into each channel of the feedforward equalizer will be moved ahead by r_{fs} samples. See [9] for more details on equalization.

In a direct adaptation DFE, the filter coefficients are computed directly from the data without imposing any sort of channel model. The least squared error (LSE) solution for the filter coefficients, using data up until time n, are given by

$$\mathbf{h}[n] = \left(\sum_{i=-\infty}^{n} \mathbf{z}[i]\mathbf{z}^H[i]\right)^{-1} \left(\sum_{i=-\infty}^{n} \mathbf{z}[i]d^*[i]\right) \qquad (3)$$

Notice that the filter coefficients now explicitly depend on time due to the dependence on the received data.

The underwater acoustic channel also has a time dependence, thus one would expect the desired equalizer coefficients to change with time. A common way to include time variation into a LSE type equalizer is to include an exponential weighting factor, $\lambda \approx 1$, into the filter coefficient equations:

$$\mathbf{h}[n] = \left(\sum_{i=-\infty}^{n} \lambda^{n-i}\mathbf{z}[i]\mathbf{z}^H[i]\right)^{-1} \left(\sum_{i=-\infty}^{n} \lambda^{n-i}\mathbf{z}[i]d^*[i]\right) \qquad (4)$$

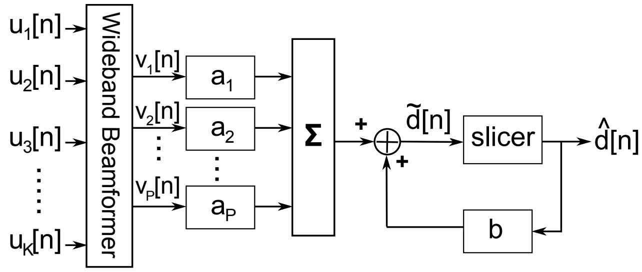

Fig. 2. A multichannel decision feedback equalizer with a beamformer front-end to reduce computational complexity.

This creates an adaptive equalizer, for which there is a common recursive algorithm called the recursive least squares (RLS) algorithm. This reduces the computational complexity considerably. For more information on adaptive filters, [10] is a commonly cited resource.

When the adaptive filter coefficients are calculated directly from the received data, the result is referred to as a direct-adaption DFE (DA-DFE). We focus on the DA-DFE structure for two reasons: first, since it has low complexity compared with the MMSE DFE (also known as the channel estimate based DFE or CEB-DFE). The complexity of the CEB-DFE is $O(L^3)$ since there is an inversion of an $L \times L$ matrix. However, using a recursive update, the DA-DFE has complexity $O(L^2)$. A second reason is that, in our experience, at a signal to noise ratio (SNR) commonly seen in underwater environments, the performance difference between the DA-DFE and the CEB-DFE is negligible.

III. Beamforming

In Eq. (4), the number of filter coefficients being estimated is $K \times (L_a + L_c) + L_{\text{fb}}$. A common engineering rule is to use the same number of feedforward coefficients as there are channel coefficients. Since the underwater channel has a long delay spread, up to hundreds of milliseconds, the computational complexity of this equalizer is very high. Stojanovic et al [2] showed that, when the signal is narrowband and the number and direction of all arrivals is known, using beamformed data is equivalent to full sensor-space processing.

Our discussion throughout this paper assumes the use of a vertical linear array. This choice is made for a number of reasons: first, it is the configuration used in the experimental data. Second, it is a common array configuration for underwater acoustics, and third, it simplifies the mathematics considerably.

Beamforming can be viewed as transforming the space in which the data is viewed, usually from physical spatial dimension to angle of arrival space. This is accomplished by applying a spatial windowing function with desired spatial-spectral characteristics. Even though underwater acoustic communication data is not narrowband, wideband beamforming methods can be used: a discrete Fourier transform (DFT) is applied first to the data, beamforming is applied separately to each frequency bin, and the inverse DFT is applied to the result. The beamforming operation can be represented mathematically as

$$\mathbf{v}(\omega) = \Phi^H(\omega)\mathbf{u}(\omega) \tag{5}$$

where $\Phi(\omega)$ is the $K \times P$ beamforming matrix at frequency ω. The notation $\mathbf{u}(\omega)$ represents the Fourier transform of the received data and $\mathbf{v}(\omega)$ represents the beamformed data, both at at frequency ω . The elements of the vectors are

$$\mathbf{u}(\omega) = \begin{bmatrix} u_1(\omega) \\ u_2(\omega) \\ \vdots \\ u_K(\omega)]^T \end{bmatrix} \qquad \mathbf{v}(\omega) = \begin{bmatrix} v_1(\omega) \\ v_2(\omega) \\ \vdots \\ v_P(\omega)]^T \end{bmatrix}$$

where $u_k(\omega)$ is the received data from sensor k and $v_p(\omega)$ is the received data in beam p. Van Trees, [11], has a much more complete description of beamforming and array processing.

After transforming the output of a wideband beamformer, $\mathbf{v}(\omega)$, into the time domain, it can be used as an input to a DA-DFE. Since the number of beams, P, is often much less than the number of sensors, K, this results in a reduction in complexity by $(P/K)^2$. Figure 2 shows the DA-DFE with a beamformer.

IV. Propagation model

Since beamforming reduces computational complexity, the next questions are how many beams to use, what shape should they be, and what direction should they point. One idea common in beamforming literature is to track each arrival separately and create an orthogonal set of beams, one for each arrival [11], [12], and [13]. These methods tend to be computationally complex and require extra adaptive algorithms since the arrivals move due to channel motion. Stojanovic [1] pointed out in her work that it is not necessary to separate the arrivals in an acoustic communication framework since the feedforward equalizer will collect the energy from all directions simultaneously.

Since channel motion induces changes in the arrival angles, our approach is to use a geometric model of the arrival structure to calculate a maximum arrival angle and use a set of beams which span that angular range.

Ray tracing is a common method used for high frequency acoustics (above 1kHz) [14]. We will use ray tracing along with the assumption that both the sea surface and sea floor are flat and parallel. This allows for a geometric model which only relies on knowledge of the water column depth, the speed of sound, the depth of the transmitter and receiver, and the distance between the transmitter and receiver. In many oceanographic applications, these measurements are readily available and change very slowly with respect to the symbol rate.

We now introduce some notation to make these ideas mathematically concrete. Table I contains the delay and elevation angle of arrival for the earliest arriving paths, using the notation:

TABLE I
TABLE OF ELEVATION ARRIVAL ANGLES AND DELAYS USING GEOMETRIC MODEL WHERE FLAT SURFACE AND BOTTOM ARE ASSUMED.

Path	Arrival Angle (in radians)	Delay (in seconds)
Direct	$\frac{\pi}{2} + \arctan\left(\frac{d_R - d_T}{\ell}\right)$	$\frac{\sqrt{(d_R-d_t)^2+\ell^2}}{c}$
Bottom	$\frac{\pi}{2} + \arctan\left(\frac{2d_w - d_R - d_T}{\ell}\right)$	$\frac{\sqrt{(2d_w-d_R-d_T)^2+\ell^2}}{c}$
Surface	$\frac{\pi}{2} - \arctan\left(\frac{d_R + d_T}{\ell}\right)$	$\frac{\sqrt{(d_R+d_T)^2+\ell^2}}{c}$
Surface-Bottom	$\frac{\pi}{2} + \arctan\left(\frac{2d_w - d_R + d_T}{\ell}\right)$	$\frac{\sqrt{(2d_w-d_R+d_T)^2+\ell^2}}{c}$
Bottom-Surface	$\frac{\pi}{2} - \arctan\left(\frac{2d_w + d_R - d_T}{\ell}\right)$	$\frac{\sqrt{(2d_w+d_T-d_T)^2+\ell^2}}{c}$

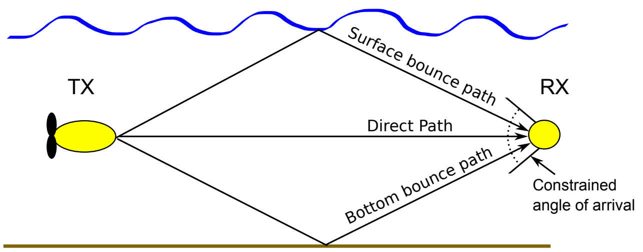

Fig. 3. Illustration of multipath and the physically constrained angle of arrival for the shallow water communications channel.

d_w water column depth [m]
d_T transmitter depth [m]
d_R receiver depth [m]
ℓ distance from transmitter to receiver [m]
c speed of sound in seawater [m/s]

According to the physics of a shallow water waveguide, there are only a finite number of propagating paths [14]. This indicates that the arrival angles are bounded within some range. Figure 3 shows an example of this when there are only three propagating paths, the direct path, the bottom bounce path, and the surface bounce path.

The next step is to extend the geometric model to allow computation of the angle of arrival for an arbitrary delay, τ_{path}. For consistency, the location of the last bounce (either surface or bottom) needs to be specified. When the last bounce is a surface bounce, the angle of arrival, θ_{path} is

$$\theta_{\text{path,surface}} = \arcsin\left(\frac{\ell}{c \cdot \tau_{\text{path}}}\right) \tag{6}$$

and when the last bounce is a bottom bounce, the angle of arrival is

$$\theta_{\text{path,bottom}} = \frac{\pi}{2} + \arccos\left(\frac{\ell}{c \cdot \tau_{\text{path}}}\right) \tag{7}$$

Using an delay, τ_{relative}, relative to the shortest path (i.e. when the propagation path is of length ℓ), the equations for angle of arrival can be rewritten. When the last bounce is a surface bounce

$$\theta_{\text{path,surface}} = \arcsin\left(\left(\frac{c \cdot \tau_{\text{relative}}}{\ell} + 1\right)^{-1}\right) \tag{8}$$

and when the last bounce is a bottom bounce

$$\theta_{\text{path,bottom}} = \frac{\pi}{2} + \arccos\left(\left(\frac{c \cdot \tau_{\text{relative}}}{\ell} + 1\right)^{-1}\right) \tag{9}$$

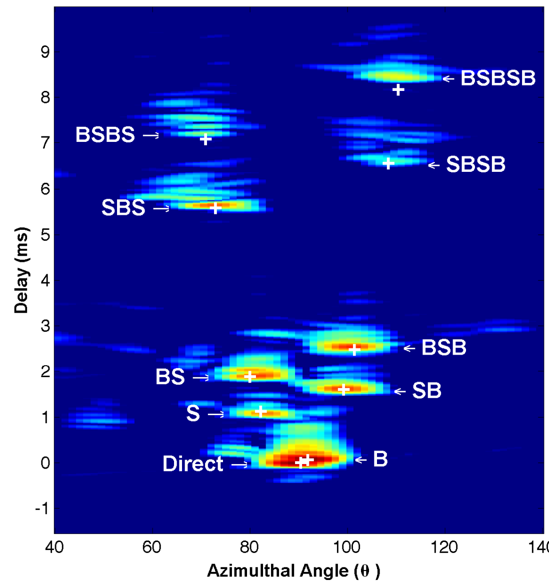

Fig. 4. Estimated angle of arrival and delay of the channel impulse response arrivals from the from the SPACE08 experiment from Julian day 290 at time 0200. The white crosses indicate the arrival points calculated from the geometrical arrival model. The arrivals are labeled according to their interaction with the surface and bottom from the transmitter to the receiver: S indicates a surface bounce and B a bottom bounce.

Figure 4 shows an estimated channel from the SPACE08 experiment which will be described in Section VI. Note that there is good agreement between the theoretical model and the actual data.

V. IMPLEMENTATION OF THE MULTICHANNEL EQUALIZER

Using the propagation model discussed in the last section, it is possible to restrict the look angles of the receiver. We devise three different methods to take advantage of this added information: a set of uniform beams spanning the angular space, a set of uniform beams time-aligned using the receiver model, and a set of discrete prolate spheroidal sequence (DPSS) beams tuned to the angles of interest.

These methods were chosen in a somewhat ad-hoc manner, but provide a way of examining whether the angle restriction can provide similar performance results to the full sensor space while providing computational advantage. It is ongoing research to find some measure of optimality for the beam patterns used.

In all of these methods, it is assumed that the delay span of the equalizer is given. Using Eq. (8) it is possible to use the delay span to calculate the maximum angle which should be considered.

A. Uniform beamformer using complete delay span

The first method we devised was to use uniformly weighted beams, placed orthogonally at the design frequency. This implies that an adjacent beam is placed at the first null of the beampattern. This provides a way both for choosing the angles and the number of beams used.

The beams are designed for a vertical linear array. The design frequency, f_d, is calculated from the element spacing of the array, d, such that, $d = f_d/(2 \cdot c)$, where c is the speed of sound in water. The first beam placed is placed at broadside since this location makes the most sense due to common communication system geometries. This also ensured as well that there were an odd number of beams and that the beams were symmetrically placed.

In order to account for the inaccuracy of the model, the delay-span of the equalizer was increased by ten symbol periods. This provided a means of helping to ensure that the motion of the individual paths did not take them beyond the maximum angle dictated by the arrival model.

B. Uniform beamformer time-aligned

The model devised in Section IV provides a way to compute the delay of first arrival is a particular look direction. Using the same set of uniform beams from before, we now time align the beams so that the delay of the first expected arrival for each of the look angles are the same. Since the model indicates there should be no energy in the beam before the first arrival, the received data before the first arrival in the beam direction is thrown out.

This provides a way to reduce the computational complexity by further reducing the number of coefficients that need to be estimated. There is some loss of energy since uniformly weighted beams are not completely confined to their look directions, so we expect there might be some small loss of performance.

The same method of padding the delay spread with ten extra symbol periods was used for this method as well. This helps doubly in this method since it helps with angle motion, and also motion in delay.

C. Discrete prolate spheroidal constraints

The main fact we are exploiting throughout this paper is the confinement of the elevation angle. Slepian [15] showed in his work that the discrete prolate spheroidal sequences (DPSS) have the maximum ratio of in band energy to out of band (or in this case in angle energy) for a given number of coefficients. The sequences are all orthogonal, and when taken together form a basis set of the sensor space. Finding the DPSS coefficients tends to be slightly computationally intensive, but this computation can be done offline or infrequently since the model parameters vary slowly.

One difference with the DPSS method over the others is that it does constrain the number of beams. A good guess of the number of beams is the number of arrivals within the angular range and delay spread used for the equalizer. This number can again be computed from the model by successively computing the arrivals until one falls outside of the desired range.

In order to handle model inaccuracies and provide a fair comparison, the maximum angle was set to be the first null of the beam with the largest look angle of the uniformly weighted beamformer method. This provided the same protection of angular motion as in previous methods.

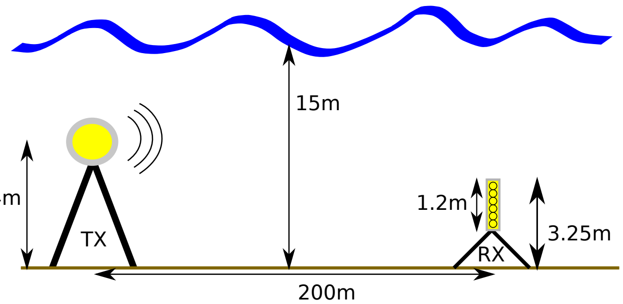

Fig. 5. Setup of SPACE08 experiment

VI. Experimental Evidence

The three proposed multichannel equalization strategies were validated using experimental evidence from the Surface Processes and Acoustic Communication Experiment (SPACE08). We used data from data sets with very different surface conditions, from a relatively calm day to a very stormy day.

A. SPACE08 Experiment setup

The SPACE08 was performed off the coast of Martha's Vineyard, MA from Oct. 14^{th} through Nov. 1^{st}. The water depth was approximately 15 meters, the transmitter was approximately 4 meters from the sea floor, and the top of the receive arrays were about 3.25 meters above the sea floor. Figure 5 illustrates the experimental setup.

The carrier frequency was $f_c = 12.5$ kHz, the bandwidth was $B = 6.51$ kHz, and the sampling frequency was $f_s = 10^7/256$. The transmitted signal was the first 20,000 symbols of a repeated binary phase shift keyed (BPSK) encoded, 4095-length M-sequence.

Before processing, the carrier was removed, the data was low-pass filtered, and the data was down-sampled to two samples per symbol. Time alignment of the signal was achieved through the use of an M-sequence timing signal at the beginning of the packet.

The receiver was a 24-element vertical array with 5 cm element spacing placed. The array was 200m from the transmitter to the southwest.

A data packet, known as an *epoch*, was transmitted once every 2 hours throughout the duration of the experiment. These data packets are referred to by the Julian day and time they were transmitted.

B. Results

In order to look at a variety of sea surface conditions, three epochs were chosen from different days: day 290 at time 0200, day 294 at time 1200, and day 300 at time 0800. These three epochs range from calm on day 290 to high stormy seas on day 300. Each of the methods described in Section V was tested for each one of these epochs, as was the full sensor-space processing and sensor-space processing using a number of sensors equal to the number of beams.

The DFE parameters were chosen such that $L_a = 20$, $L_c = 80$ and $L_{\text{fb}} = 99$. This captured most of the multipath signal

TABLE II
INPUT SIGNAL TO NOISE RATIO FOR EACH OF THE DATA EPOCHS EXAMINED FROM THE SPACE08 EXPERIMENT.

Epoch	Input SNR
Day 290, time 0200	35.5 dB
Day 294, time 1200	34.7 dB
Day 300, time 0800	34.1 dB

energy in the feedforward equalizer and canceled most of the ISI in the feedback equalizer.

An RLS algorithm was used to estimate the DA-DFE equalizer taps. In order to ensure that the results we observed were not artifacts of the choice of the exponential weighting factor, λ, ten different values were tested from $\lambda = 0.991$ to $\lambda = 0.9999$. The DPSS trials on day 290 and 300 varied from this convention and were tested on nine values from $\lambda = 0.996$ to $\lambda = 0.9999$.

For the geometry of the experiment setup, our arrival model indicated that 7 uniformly weighted beams was appropriate, and also coincidentally that there were 7 arrivals within the examined delay and angular spread. Thus, for all the proposed methods, 7 beams was appropriate. The elevation angles examined for the beamspace methods were from 75.5^o to 104.5^o. For the DPSS method, the angular spread used was 70.3^o to 109.7^o.

The complexity of both the DPSS method and the beamspace method were the same: both were a factor of $(24/7)^2 \approx 11.75$ times less complex than the full sensor space processing. The time aligned processing used 532 equalizer coefficients, which reduces the complexity by $(700/532)^2 \approx 1.75$ over the other proposed methods and a factor of 20.35 over full beamspace processing.

The measure used for comparing the different methods is the output soft decision error (SDE), ϵ_{SDE}. This is a measure of the residual error after equalization and can be represented mathematically as

$$\epsilon_{\mathrm{SDE}} = \frac{1}{N} \sum_{n=1}^{N} \frac{|d[n] - \tilde{d}[n]|^2}{|d[n]|^2} \tag{10}$$

where $d[n]$ is the transmitted symbol, and $\tilde{d}[n]$ is the filtered data before the symbol decision. In all cases, the bit error rate (BER) was equal to zero due to the high operating SNR.

Figure 6 shows the results from the day 290, time 0200 epoch, Figure 7 shows the results from the day 294, time 1200 epoch, and Figure 8 shows the results from the day 300, time 0800 epoch. The input SNR is the ratio of the measured incoherent signal energy to noise energy before equalization. The observed input SNR for each epoch is given in Table II.

The results for the proposed methods are similar across all of the epochs: the DPSS method and the beamspace method perform nearly equivalently and do as well (as on day 290) or nearly as well (the other two days) as the full beamspace processing; the results are within 1 dB of one another. As expected, the time-aligned beamspace method does slightly worse, but again it is within 1.5 dB of the best method.

For comparison of computationally similar methods, seven of the twenty-four sensors were used as the input to a multichannel equalizer. Several configurations of the seven sensors were tested and the best configuration for each epoch is shown on the figures. In all cases, the best seven sensors perform at least 2 dB worse than either the proposed methods or the full sensor space. Thus, for the same computational complexity, the proposed methods improve the performance dramatically and compete quite favorable with the full sensor space results.

All of the results presented depend heavily on the choice of the exponential weighting factor, λ. Finding the optimal λ for a given channel is beyond the scope of this work, but it should be kept in mind that any results presented which use an RLS algorithm are going to be sensitive to this choice.

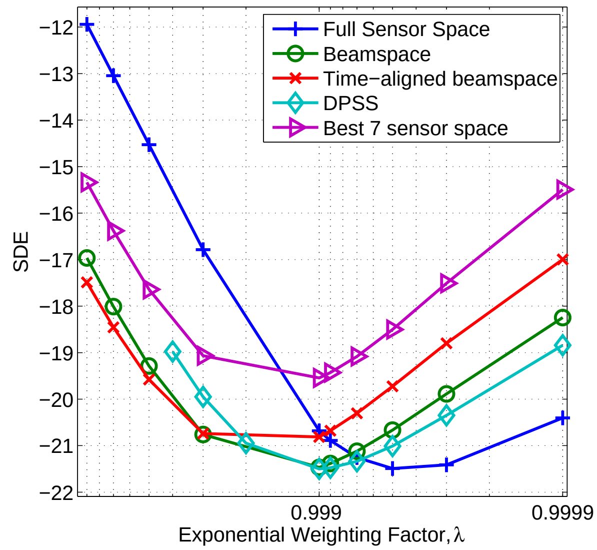

Fig. 6. Results from SPACE08 experiment for day 290, time 0200.

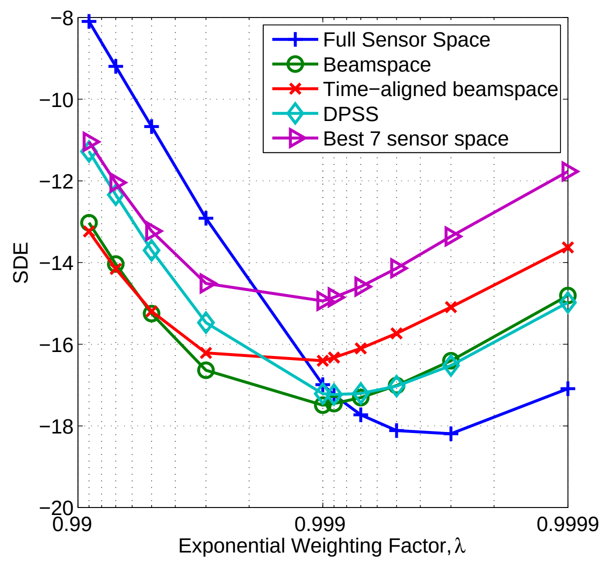

Fig. 7. Results from SPACE08 experiment for day 294, time 1200.

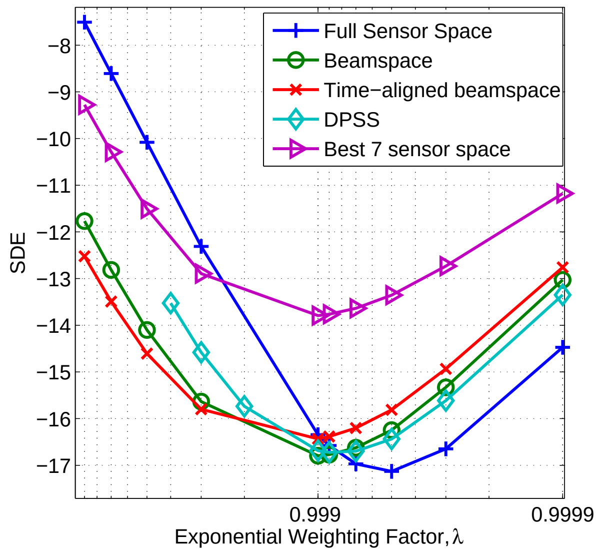

Fig. 8. Results from SPACE08 experiment for day 300, time 0800.

Results in the past have compared methods for a particular value of λ, which can lead to deceptive results.

VII. Conclusion

In this paper we presented a physically motivated waveguide constraint for reducing the degrees of freedom for a multichannel equalizer used in an underwater acoustic communication system. The waveguide constraints are based on a simple model of the arrival structure which can easily be constructed with commonly available sensors.

Using this method we were able to reduce the computational complexity by as much as a factor of 20 without sacrificing more than 1.5 dB of residual mean squared error performance. All of the proposed methods appear to be robust to sea surface conditions such the their relative performance is maintained even when the sea surface is more variable. Of the three proposed methods, beamspace processing with full delay spread, time-aligned beamspace processing, and DPSS beam processing, beamspace processing with full delay spread has the best residual error on the SPACE08 data sets while still reducing the complexity by eleven fold over the complete sensor space processing.

When the best set of sensors are chosen such that the computational complexity would be the same for the proposed reduced-complexity methods, the proposed methods perform up to 3 dB better than sensor space processing.

The reduced complexity processing performed better when the modeling assumptions were met, i.e. when there was less channel motion so the estimated angular spread was approximately correct. However, even when these assumptions were not perfectly met, the experimental evidence gave at most 1 dB difference between the reduced-complexity and the full sensor-space processing.

The proposed DPSS method also presents a different way to view beamspace processing: as spanning a space rather than steering beams towards arrivals. This is the first work the authors know of using this alternative view.

This work represents a first step toward incorporating physical knowledge into equalization algorithms. The proposed methods are somewhat ad-hoc, but work well on the experimental data. Further work is needed to find ways to optimally include physical models into underwater communication systems.

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