EFFECTIVE NOISE CORRELATION MATRIX STRUCTURE FOR EQUALIZATION OF SHALLOW WATER CHANNELS

Ballard J. S. Blair and James C. Preisig

Dept. of Applied Ocean Physics and Engineering
Woods Hole Oceanographic Institution
Woods Hole, MA 02543 USA

ABSTRACT

Shallow water acoustic communication is challenging due to the delay and Doppler spread resulting from acoustic scattering from surface gravity waves. A channel estimate based decision feedback equalizer (CEB-DFE) has been shown to be very effective at mitigating these channel effects. One component of the DFE that is often overlooked is the effective noise correlation matrix. In much of the literature, the effective noise correlation matrix is approximated by a scaled identity matrix, where the scaling is assumed to be near the reciprocal of the signal to noise ratio (SNR). For the underwater channel, explicitly estimating the full effective noise correlation matrix leads to a reduction of the residual data estimation error. In this paper we show that correlated changes in channel impulse response coefficients cause the effective noise correlation matrix to have off-diagonal terms. Since the correlated changes tend to occur slowly over time, the effective noise correlation matrix is Toeplitz. An algorithm which exploits this fact to reduce computational complexity is presented and is demonstrated using experimental data.

Index Terms— digital communication, acoustics, equalization, noise

I. INTRODUCTION

The shallow water acoustic communication problem is challenging since the channel is time-varying, delay-spread, and highly band-limited [1]. The use of a decision feedback equalizer (DFE) is a common method in the literature for handling the adversities of the channel [2], [3]. It has been previously shown that a channel estimate based DFE (CEB-DFE) outperforms one where the equalizer coefficients are directly adapted from the received data [4].

What is often overlooked in the CEB-DFE formulation is that here are two quantities needed to calculate the equalizer coefficients: an estimate of the channel impulse response and an estimate of the effective noise correlation matrix [5]. This second quantity is often approximated as a scaled identity matrix with a scaling equal to the reciprocal of the signal to noise ration (SNR). [4], [6]. For the underwater channel, this turns out to be a poor estimate of the effective noise correlation matrix and it has been shown that estimating the full matrix reduces the residual data estimation error [7].

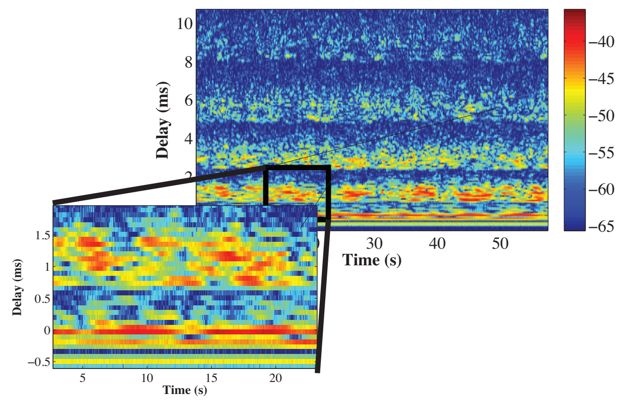


Fig. 1. The magnitude of an estimated channel impulse response at 1km from the transmitter (SPACE08 experiment).

In the underwater environment, neighboring channel coefficients are often correlated [1]. Figure 1 shows an example of a measured time-varying impulse response from the surface processes and acoustic communication experiment (SPACE08) in 2008. Notice that neighboring channel impulse response coefficients appear to have correlation time variation.

In this work we will show that it is this correlated variation that is responsible for the effective noise correlation matrix having a non-diagonal structure. Furthermore we present a computationally efficient algorithm for calculating the effective noise correlation matrix.

The remainder of this paper will be as follows: Section II presents the model used for the underwater channel. Section III describes the structure of the CEB-DFE. The physical explanation for the structure of the effective noise correlation matrix is given in IV. The algorithm for efficiently estimating the effective noise correlation matrix is given in Section V and experimental results demonstrating its use are given in VI. Lastly, Section VII presents a summary and concluding thoughts.

Throughout this paper, lowercase bold letters, e.g. $\mathbf{a}$, indicate vectors, uppercase bold letters, e.g. $\mathbf{A}$, indicate matrices, and non-bold symbols are used for scalars. The symbols ^{T} indicates the transpose of a quantity, * the conjugate, and ^{H} the conjugate transpose or Hermitian. All vectors are assumed to be column vectors.

II. CHANNEL MODEL

The underwater communication channel is well modeled as a linear time-varying channel plus (possibly correlated) Gaussian noise. A vector of received data, $\mathbf{u}[n]$ can be written in matrix vector form as [7]:

$$\mathbf{u}[n] = \mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] \tag{1}$$

where $\mathbf{d}[n]$ is a vector of transmitted data symbols with energy $\mathrm{E}\{|d[n]|^2\} = \sigma_d^2$, and $\mathbf{v}[n]$ is noise modeled using a vector of samples from a wide sense stationary (WSS) zero-mean random process with variance $\mathbf{R}_v$. The elements of the transmitted data, received data, and noise vectors are:

$$\mathbf{u}[n] = [u[n-L_c+1] \ \ldots \ u[n] \ \ldots \ u[n+L_a]]^T \tag{2}$$

$$\mathbf{d}[n] = [d[n-L_c-N_c+2] \ \ldots \ d[L_a+N_a]]^T \tag{3}$$

$$\mathbf{v}[n] = [v[n-L_c+1] \ \ldots \ v[n] \ \ldots \ v[n+L_a]]^T \tag{4}$$

where L_a and L_c are the number of acausal and causal taps respectively in the feedforward section of the MMSE DFE equalizer and N_a and N_c are the number of causal and acausal coefficients in the channel impulse response.

The rows of the matrix $\mathbf{G}[n]$ are the time-varying channel impulse response coefficients, padded with zeros so the matrix product $\mathbf{G}[n]\mathbf{d}[n]$ is equivalent to the convolution of the channel impulse response with the transmitted data. The matrix $\mathbf{G}[n]$ will be hereafter referred to as the channel convolution matrix. In this paper, symbol-spaced sampling is assumed for clarity, but extension to fractionally-spaced sampling is straight forward.

The actual values of the channel convolution matrix are rarely known *a-priori* and must be estimated from the received data along with either a known training sequence or past data estimates. This work follows the common engineering practice of assuming the channel is time invariant for the delay spanned by the channel convolution matrix. Therefore, one only one channel estimate is used to populate the whole matrix.

A recursive least squares (RLS) algorithm is used in this work to estimate the channel impulse response due to its effectiveness and reasonable computational complexity of order $O(N^2)$. This algorithm tracks the channel impulse response coefficients as they vary in time, but does not require structured knowledge of the variation as is required for Kalman filtering and related methods. The estimate of the channel impulse response coefficients using an RLS approach are:

$$\widehat{\mathbf{g}}[n] = \left(\sum_{i=-\infty}^{n} \lambda^{n-i}\mathbf{d}'[i]\mathbf{d}'^H[i]\right)^{-1}\left(\sum_{i=-\infty}^{n} \lambda^{n-i}\mathbf{d}'[i]u^*[i]\right) \tag{5}$$

where $\mathbf{d}'[n]$ is a data vector of length N equal to the delay spread of the channel, $N = N_a + N_c$, and $\lambda \approx 1$ is an exponential weighting factor. The exact derivation of the RLS algorithm is outside the scope of this work, but the details are readily available in the literature (e.g. [8] is a commonly cited resource).

III. CHANNEL ESTIMATE BASED DECISION FEEDBACK EQUALIZATION

The decision feedback equalizer (DFE) is has been used widely in the underwater environment, even though it is not optimal, because it provides a computationally tractable way to mitigate channel effects [3]. The DFE consists of two linear filters working in concert: the feedforward filter collects the energy from the received signal and shapes its response and the feedback filter cancels the inter-symbol interference (ISI) from previously received symbols [9]. The general DFE equation can be written as:

$$\widetilde{d}[n] = \sum_{\ell=-L_a}^{L_c-1} h_{\mathrm{ff}}^*[\ell]u[n-\ell] + \sum_{k=1}^{L_{\mathrm{fb}}} h_{\mathrm{fb}}^*[k]\widehat{d}[n-k] \tag{6}$$

where $u[n]$ is the baseband received data, $\widehat{d}[n]$ is the past symbol decisions, and $\widetilde{d}[n]$ is the filtered received data before a symbol decision has been made. The feedforward filter coefficients are represented as $h_{\mathrm{ff}}[n]$ and the feedback coefficients as $h_{\mathrm{fb}}[n]$. The total number of DFE coefficients is $L = L_a + L_c + L_{\mathrm{fb}}$.

The DFE equation can be represented more compactly using vector notation as:

$$\widetilde{d}[n] = \mathbf{h}_{\mathrm{ff}}{}^H\mathbf{u}[n] + \mathbf{h}_{\mathrm{fb}}{}^H\widehat{\mathbf{d}}[n-\ell] = \mathbf{h}^H\mathbf{z}[n] \tag{7}$$

where $\mathbf{h}^T = [\mathbf{h}_{\mathrm{ff}}{}^T \ \ \mathbf{h}_{\mathrm{fb}}{}^T]$ is a vector of filter coefficients and $\mathbf{z}^T[n] = [\mathbf{u}^T[n] \ \ \widehat{\mathbf{d}}^T[n]]$ is a data vector containing both the received data and the past symbol estimates.

A common cost criterion to find the optimal filter coefficients is the mean squared error between the transmitted data and the filtered received data, $\widetilde{d}[n]$, written as:

$$J(\mathbf{h}) = \mathrm{E}\{|d[n] - \widetilde{d}[n]|^2\} = \mathrm{E}\{|d[n] - \mathbf{h}^H\mathbf{z}[n]|^2\} \tag{8}$$

Minimizing this cost criterion across all filter coefficients gives the Weiner-Hopf equation for the optimal coefficients:

$$\widehat{\mathbf{h}} = \left(\mathrm{E}\{\mathbf{z}[n]\mathbf{z}^H[n]\}\right)^{-1}\left(\mathrm{E}\{\mathbf{z}[n]d^*[n]\}\right) \tag{9}$$

Now, using the model from Eq. (1), the vector $\mathbf{z}[n]$ can be rewritten as:

$$\mathbf{z}[n] = \begin{bmatrix}\mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] \\ \widehat{\mathbf{d}}[n]\end{bmatrix} \tag{10}$$

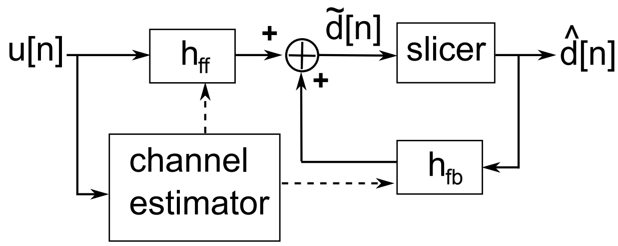


Fig. 2. Illustration of the structure of a CEB-DFE.

At this point, it is useful to introduce some new notation. The columns of the channel convolution matrix, $\mathbf{G}[n]$, can be labeled according to offset of the data symbol by which they are multiplied relative to the data symbol being estimated:

$$\mathbf{G}[n] = \left[\mathbf{g}_{-L_c-N_c+2}[n] \;\; \ldots \;\; \mathbf{g}_0[n] \;\; \ldots \;\; \mathbf{g}_{L_a+N_a}[n]\right] \quad (11)$$

where the column $\mathbf{g}_0[n]$ corresponds to the data symbol that is being estimated at time n. A new matrix, $\mathbf{G}_{\text{fb}}[n]$, is assembled from the columns corresponding to the data symbols used in the feedback filter of the DFE. The remaining columns are assembled into another new matrix, $\mathbf{G}_o[n]$. If all previously received data symbols are used in the feedback filter, the channel convolution matrix can be written as:

$$\mathbf{G}[n] = \left[\mathbf{G}_{\text{fb}}[n] \quad \mathbf{G}_0[n]\right] \quad (12)$$

Plugging in Eq. (10) into Eq. (9) and assuming the channel convolution matrix is known, the MMSE DFE coefficients are given by the expressions [7]:

$$\begin{aligned} \mathbf{h}_{\mathbf{ff}}[n] &= (\mathbf{G}_0[n]\mathbf{G}_0^H[n] + \sigma_d^{-2}\mathbf{R}_{\mathbf{v}})^{-1}\mathbf{g}_0 & (13) \\ \mathbf{h}_{\mathbf{fb}}[n] &= -\mathbf{G}_{\text{fb}}{}^H[n]\mathbf{h}_{\mathbf{ff}}[n] & (14) \end{aligned}$$

When the channel convolution matrix is not known, it is replaced by an estimate, $\widehat{\mathbf{G}}[n]$. This version of the decision feedback equalizer is known as the channel estimate based decision feedback equalizer (CEB-DFE) for obvious reasons. Figure 2 shows a block diagram of the structure of this equalizer.

It is often assumed for terrestrial RF communication systems that the observation noise, $\mathbf{v}[n]$, is white with zero mean and variance σ_v^2 [6]. This implies that the noise correlation matrix, $\mathbf{R}_{\mathbf{v}}$ is a scaled identity matrix, such that $\mathbf{R}_{\mathbf{v}} = \rho\mathbf{I}$, where the inverse SNR $\rho = \sigma_v^2/\sigma_d^2$ [4]. It has previously been shown that this assumption leads to an increased residual data estimation error for underwater communications [7]. The next section provides physical reasoning for why the assumption of a diagonal noise correlations matrix is not correct for the underwater channel.

IV. STRUCTURE OF EFFECTIVE NOISE CORRELATION MATRIX

In underwater communication systems the channel coefficients are rarely known *a-priori* and must be estimated from the received data. Due to observation noise and the time-variability of the channel, the estimate of the channel usually contains some error. This estimation error can be represented mathematically as:

$$\mathbf{G}[n] = \widehat{\mathbf{G}}[n] + \Gamma[n] \quad (15)$$

where $\widehat{\mathbf{G}}[n]$ is the estimate of the channel convolution matrix and $\Gamma[n]$ is the error in the estimate. The received data vector can be rewritten using this model:

$$\begin{aligned} \mathbf{u}[n] &= \mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] \\ &= \widehat{\mathbf{G}}[n]\mathbf{d}[n] + \Gamma[n]\mathbf{d}[n] + \mathbf{v}[n] \\ &= \widehat{\mathbf{G}}[n]\mathbf{d}[n] + \mathbf{w}[n] \end{aligned} \quad (16)$$

where the effective noise, $\mathbf{w}[n] = \Gamma^H[n]\mathbf{d}[n] + \mathbf{v}[n]$, includes the portion of the received signal that is not included in the term $\widehat{\mathbf{G}}[n]$. Assuming that the transmitted data symbols are independent and have unit energy ($\sigma_d^2 = 1$), the variance of the effective observation noise is a sum of the observation noise variance and a term that depends on the channel estimation errors. This can be written as:

$$\begin{aligned} \mathbf{R}_{\mathbf{W}}[n] &= \mathrm{E}\{\mathbf{w}[n]\mathbf{w}^H[n]\} \\ &= \mathrm{E}\{(\Gamma[n]\mathbf{d}[n] + \mathbf{v}[n])(\Gamma[n]\mathbf{d}[n] + \mathbf{v}[n])^H\} \\ &= \mathbf{R}_\Gamma[n] + \mathbf{R}_{\mathbf{v}}[n] \end{aligned} \quad (17)$$

where $\mathbf{R}_\Gamma[n] = \mathrm{E}\{\Gamma[n]\Gamma^H[n]\}$ is the channel estimation error correlation matrix.

Assuming that the MMSE channel estimate is used, so the error is zero-mean (the estimator is unbiased) and uncorrelated with the estimator, the feedforward and feedback DFE equalizer coefficients can be written as:

$$\begin{aligned} \mathbf{h}_{\mathbf{ff}}[n] &= (\widehat{\mathbf{G}}_0[n]\widehat{\mathbf{G}}_0^H[n] + \mathbf{R}_\Gamma[n] + \sigma_d^{-2}\mathbf{R}_{\mathbf{v}}[n])^{-1}\mathbf{g}_0 \\ \mathbf{h}_{\mathbf{fb}}[n] &= -\widehat{\mathbf{G}}_{\text{fb}}{}^H[n]\mathbf{h}_{\mathbf{ff}}[n] \end{aligned} \quad (18)$$

The feedback equalizer coefficients have the same form as before with the estimate used in place of the true channel coefficients. An additional term has appeared in the feedforward equalizer coefficients that is a product of the channel convolution matrix error terms. This error term and the true noise correlation matrix are grouped into one term, which will be called the effective noise correlation matrix, $\mathbf{R}_0$. This matrix is the sum of the interference caused by channel estimation errors and the scaled observation noise.

$$\mathbf{R}_0 = \mathbf{R}_\Gamma[n] + \sigma_d^{-2}\mathbf{R}_{\mathbf{v}}[n] \quad (19)$$

The elements of $\mathbf{R}_\Gamma[n]$ are related to the correlation between the channel estimation error coefficients. The estimation error in the channel is a result of both observation noise and the fact that the estimate is found by an averaging method which causes a lag error [10]. The optimal point of channel estimation is where the trade-off between lag error and observation noise induced error is a minimum: longer observation windows lead to more lag error but decrease observation noise induced error.

The main diagonal of the matrix $\mathbf{R}_\Gamma[n]$ is the total energy in the channel estimation error and the off-diagonal terms represent the correlation across time and delay of the channel estimation errors. If there is correlated motion in the channel impulse response coefficients, there will be correlated lag error across the coefficients. This leads to off-diagonal terms at locations corresponding to the delay between the channel impulse response coefficients: often neighboring coefficients are more correlated leading to larger values near the main diagonal of $\mathbf{R}_\Gamma[n]$.

V. ESTIMATING THE EFFECTIVE NOISE CORRELATION MATRIX

In the underwater environment, the effective noise correlation is much smaller on a term by term basis than the term induced by the channel estimation errors. Therefore, it is the structure of this channel estimation error term that is important. The correlation structure of the channel errors are slowly varying compared with common symbol rates (typically around five kilo-symbols per second or higher). The effective correlation matrix is therefore well approximated by a Toeplitz-Hermitian matrix. Tracking only the first row (or column) of the matrix is sufficient to create the whole structure of the matrix.

An implicit assumption of this approach is that the channel is roughly time-invariant for the delay spanned by the channel convolution matrix. If this were not the case, there would be more noise in the estimates of the bottom rows of $\widehat{\mathbf{G}}[n]$ than the top, which would cause the matrix not to be Toeplitz. The results from the data examined seem to imply that the assumption of time-invariance is reasonable, but this effect might be important in channels with very high Doppler.

Another assumption of this approach is that the correlation across delay is approximately time-invariant on the sampling timescale. This is a safe assumption since processes that are changing more rapidly than the sampling time appear as incoherent noise in the output. Also, averaging methods are used to estimate the effective noise correlation matrix, so transient effects will not strongly influence the estimate.

The error vector is calculated using the channel estimate, estimates of past transmitted data, and the received data [7].

$$\widehat{\mathbf{e}}[n] = \mathbf{u}[n] - \widehat{\mathbf{G}}[n]\widehat{\mathbf{d}}[n] \tag{20}$$

Using the channel model from Eq. (1), the error vector can be rewritten as [7]:

$$\widehat{\mathbf{e}}[n] = \mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] - \widehat{\mathbf{G}}[n]\mathbf{d}[n] = \Gamma[n]\mathbf{d}[n] + \mathbf{v}[n] \tag{21}$$

In order to maximize the number of snapshots available, the unbiased correlation of the error vector is calculated as:

$$\gamma_i[n] = \frac{1}{L-1-i} \sum_{j=0}^{L-1-i} \widehat{e}[n, i+j]\widehat{e}^*[n, j], \quad i = 0, \ldots, L-1 \tag{22}$$

where $\widehat{e}[n,k]$ is the kth element of the error vector at time n, $L = L_c + L_a$ is the number of equalizer coefficients, and γ_i is the i^{th} component of the unbiased error vector correlation. The $\gamma_i[n]$ values are averaged using a exponentially decreasing window to estimate the values of the first row of the effective noise correlation matrix:

$$\widehat{\mathbf{R}}_{0,[1,i]}[n] = \sum_{k=0}^{n} \lambda\gamma_i[k] \tag{23}$$

This allows the algorithm to capture the time-varying nature of the statistics while approximating the value $R_0[1,i]$, the i^{th} component of the first row of the effective noise correlation matrix at time n. The complete effective noise correlation matrix is constructed using assuming a Toeplitz, Hermitian structure of the effective noise correlation matrix.

There are several advantages to this algorithm over previously proposed algorithms:

- It reduces the number of components that must be tracked to L from L^2. This helps with the space requirements for the algorithm and allows for extra averaging of the quantities.
- It introduces an easy flexibility for modifying the structure of the interference plus noise correlation matrix. If it is assumed that the matrix is tri-diagonal, then only two parameters have to be tracked. The constructed matrix will be tri-diagonal, with the remaining elements all set to zero.
- If this structure is valid, there may be a performance gain due to the extra averaging of the error coefficients. Since there are fewer values to track, it is possible to track them with lower error.

The last point is especially interesting since underwater communication problems are often data limited due to time-variation of the channel. This method provides a way to more effectively use the available data. In the next section, this algorithm is applied to experimental data and compared with other approaches.

VI. EXPERIMENTAL RESULTS

The SPACE08 was performed off the coast of Martha's Vineyard, MA from Oct. 14$^{\text{th}}$ through Nov. 1$^{\text{st}}$. The water depth was approximately 15 meters, the transmitter was approximately 4 meters from the sea floor, and the bottom of the receive arrays were about 3.25 meters above the sea floor. Figure 3 illustrates the setup of this experiment.

The data signal had a bandwidth of $B = 6.51$ kHz and was modulated onto a carrier with frequency $f_c = 12.5$ kHz. The sampling frequency was $f_s = 10^7/256$. The transmitted signal analyzed here is a 4095-length M-sequence that was repeated 89 times for a packet that is one minute in length (with some zero-padding). The data was modulated using binary phase shift keying (BPSK) onto a square-root raised cosine pulse.

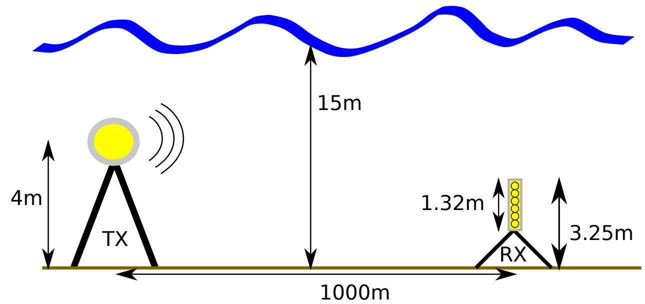


Fig. 3. Setup of the SPACE08 experiment for the 1 km receiver.

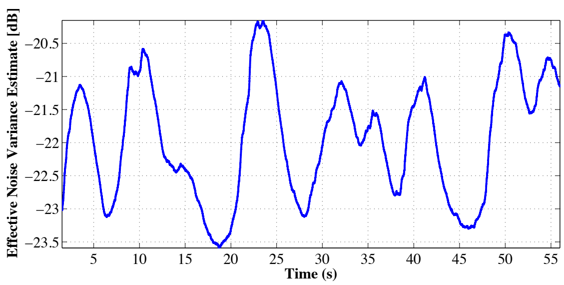


Fig. 4. Top-left element of the $\widehat{\mathbf{R}}_0$ matrix as tracked using the method proposed in [7]. This value is a measure of the effective noise variance.

Figure 4 shows a plot of the top-left element of the effective noise correlation matrix using an CEB-DFE algorithm where the entire structure of the effective noise correlation matrix is estimated. This data is from the third element from the top of the receive array located 1 km to the southeast of the transmitter. This plot shows emphasizes the fact that the effective noise autocorrelation matrix is *not* time-invariant, there are swings of 4dB, and must be tracked in order for good equalizer performance.

Several methods for approximating the effective noise correlation matrix were examined. Table I provides a description of each of the methods examined.

Figure 5 shows the bit error rate and soft decision error from the SPACE08 experiment. The soft decision error is the residual data estimation error before any symbol decisions are made and is given by:

$$\epsilon_{\text{SDE}} = \frac{\sum |d[n] - \widetilde{d}[n]|^2}{\sigma_d^2} \tag{24}$$

The SNR is varied by adding in the appropriate amount of noise which was measured using the same hydrophone during the same time period.

The data clearly shows that there is a penalty for assuming that the estimated noise matrix is diagonal (labeled *DIAG* in the plot). However, there is no additional penalty for estimating this diagonal matrix using only one estimated value and assuming it is Toeplitz (labeled *SING* in this plot).

Table I. Description of methods compared using the SPACE08 data set.

Method Label	Description
CEB	CEB-DFE where the full effective noise correlation matrix is estimated from the data.
DA	DFE where the equalizer coefficients are estimated directly from the data (direct adaptation).
DIAG	CEB-DFE where first entry (top-right entry) of the matrix $\widehat{\mathbf{R}}_0$ is used as an estimate of the effective noise variance. The effective noise correlation matrix is approximated as a scaled identity matrix.
SING	CEB-DFE where the effective noise correlation matrix is approximated by a diagonal matrix with entries equal to the main diagonal of $\widehat{\mathbf{R}}_0$.
CALC	CEB-DFE where the effective noise correlation matrix is estimated as a Topelitz-Hermitian matrix as described in this paper.
AMB	CEB-DFE where the effective noise correlation matrix is approximated as a scaled identity matrix, where the scaling is based on the SNR measured from the basebanded data before any equalization.

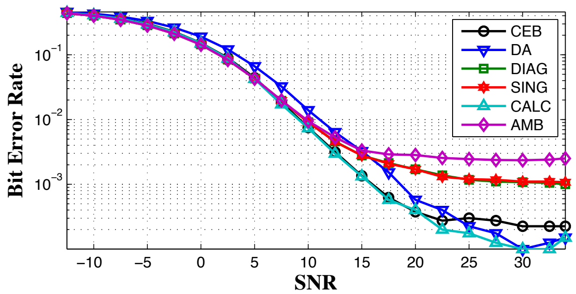
By directly adapting the equalizer coefficients from the data (labeled *DA*), the performance is similar at high SNR to the MMSE and proposed algorithm. The performance falls off as the noise is increased. This is likely due to the noise being non-stationary and so the variation of the noise negatively affects the direct adaptation more than the channel estimate based algorithms.

Overall, this data shows that the proposed method for estimating the effective noise correlation matrix as a Toeplitz-Hermitian matrix (labeled *CALC*) performs as well as algorithm which estimates the complete effective noise correlation matrix (labeled *CEB*). This is a win computationally since the proposed method is linear in the dimension of the effective noise correlation matrix, while the previous method is quadratic. Also, it demonstrates that the assumption that the channel is roughly time-invariant over the delay of the channel convolution matrix is valid for this data set since the Toeplitz-Hermitian matrix was a good approximation.

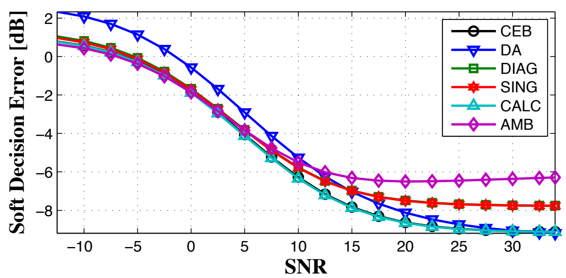
VII. CONCLUSIONS

In this paper we have provided a physical explanation for the existence on off-diagonal elements in the effective noise correlation matrix: neighboring channel impulse response coefficients vary in a correlated manor. This insight explains why the off-diagonal terms appear for underwater acoustic communication systems, but aren't present in terrestrial RF communication systems.

An algorithm for efficiently estimating the effective channel correlation matrix was provided that did as well as any previously proposed algorithm, but at a lower computational cost. This algorithm took advantage of the fact that the



(a) Bit error rate (BER) results



(b) Soft decision error (SDE) results

Fig. 5. Results after equalizing 1000m data from the SPACE08 experiment. CEB is the MMSE DFE where the full effective noise correlation matrix is estimated, DA is the direct adaptation DFE, DIAG is the MMSE DFE only using the diagonal portion of $\widehat{\mathbf{R}}_0$, SING is the MMSE DFE only using the top-left entry of $\widehat{\mathbf{R}}_0$ times the identity matrix, CALC is the proposed MMSE algorithm, and AMB is the MMSE DFE where the averaged noise variance times the identity matrix is used.

effective noise statistics are slowly varying and so the matrix is well approximated as Toeplitz-Hermitian. The reduction in computational complexity is important in array systems where the number of coefficients being estimated is quite large and efficient algorithms are needed for practical implementation.

Also, it was shown that at the least, the effective noise variance needs to be continuously tracked throughout a data packet in order to prevent loss of system performance. Since much of the literature uses one estimate of the effective noise variance, this is another way in which underwater systems differentiate themselves from their terrestrial counterparts.

Acknowledgments

The authors would like to thank the office of naval research for their continued support, in particular for their support of this research through grants: ONR Grant #N00014-05-10085, ONR Grant #N00014-07-10184, and ONR Grant # N00014-09-1-0540.

The authors would also like to thank Dr. Andrew Singer from the University of Illinois Urbana-Champaign for sparking the first inquiry into this line of research and Dr. Milica Stojanovic from Northeastern University for her help flushing out the ideas in this paper.

VIII. REFERENCES

[1] M. Stojanovic, "Underwater acoustic communications: Design considerations on the physical layer," in *Wireless on Demand Network Systems and Services, 2008. WONS 2008. Fifth Annual Conference on*, 2008, pp. 1 –10.

[2] M. Stojanovic, J. Catipovic, and J. Proakis, "Adaptive multichannel combining and equalization for underwater acoustic communications," *Journal of the Acoustical Society of America*, vol. 94, no. 3, pp. 1621–1631, Sep. 1993.

[3] M. Stojanovic, "Recent advances in high-speed underwater acoustic communications," *IEEE Journal of Oceanic Engineering*, vol. 21, no. 2, pp. 125–136, 1996.

[4] P. K. Shukla and L. F. Turner, "Channel-estimation-based adaptive DFE for fading multipath radio channels," *IEE Proceedings In Communications, Speech and Vision*, vol. 138, no. 6, pp. 525–543, 1991.

[5] S. Qureshi, "Adaptive equalization," *Proc. of the IEEE*, vol. 73, no. 9, pp. 1349–1387, Sep. 1985.

[6] J. Proakis, *Digital Communications*, 4th ed. McGraw-Hill, 2000.

[7] J. Preisig, "Performance analysis of adaptive equalization for coherent acoustic communications in the time-varying ocean environment," *J. Acoust. Soc. Amer.*, vol. 118, pp. 263–278, Jul. 2005.

[8] S. Haykin, *Adaptive Filter Theory*, 4th ed. Prentice Hall, 2001.

[9] M. Lopez, A. Singer, S. Whitney, and G. Edelson, "A DFE coefficient placement algorithm for underwater digital acoustic communications," in *Oceans '99. MTS/IEEE. Riding the Crest into the 21st Century. Conference and Exhibition. Conference Proceedings (IEEE Cat. No.99CH37008)*, Seattle, WA, USA, Sep. 1999, pp. 996–1001.

[10] E. Eleftheriou and D. D. Falconer, "Tracking properties and Steady-State performance of RLS adaptive filter algorithms," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, pp. 1097–1110, Oct. 1986.