Soft Channel Estimation

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Typical Scattering Function

Depth = 15m
Distance = 225m

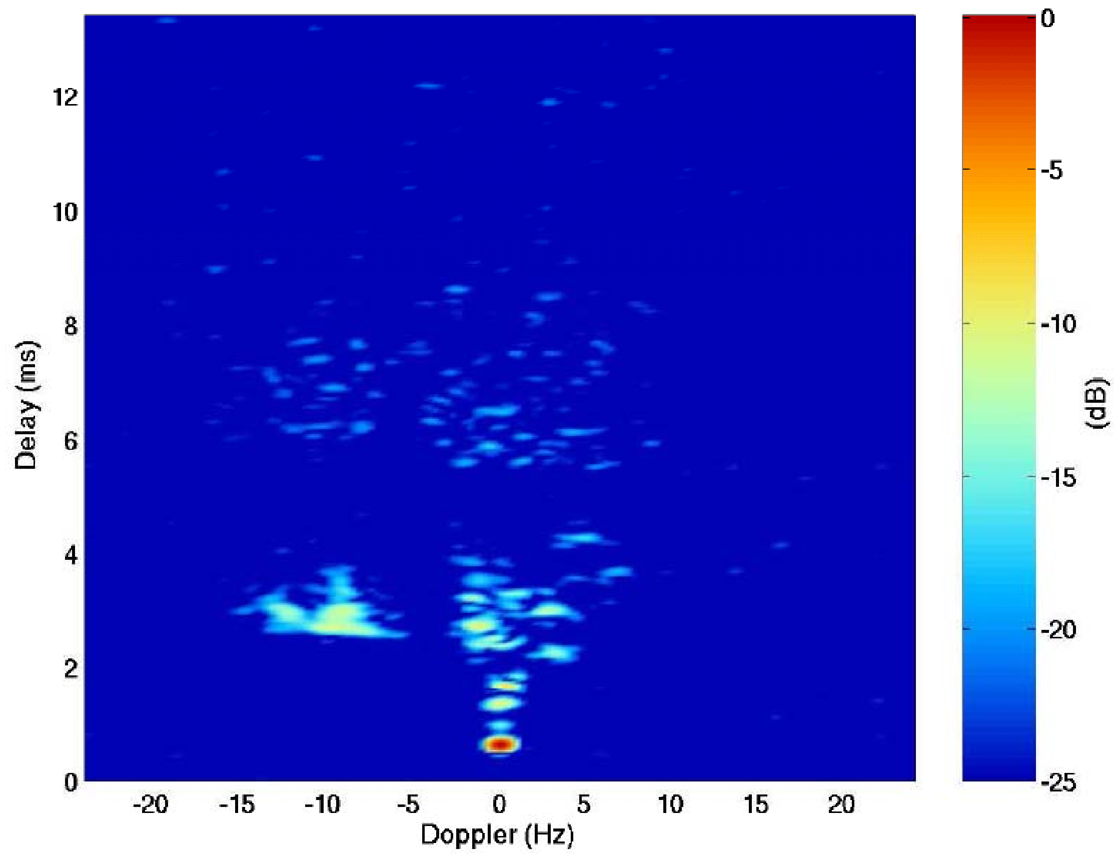
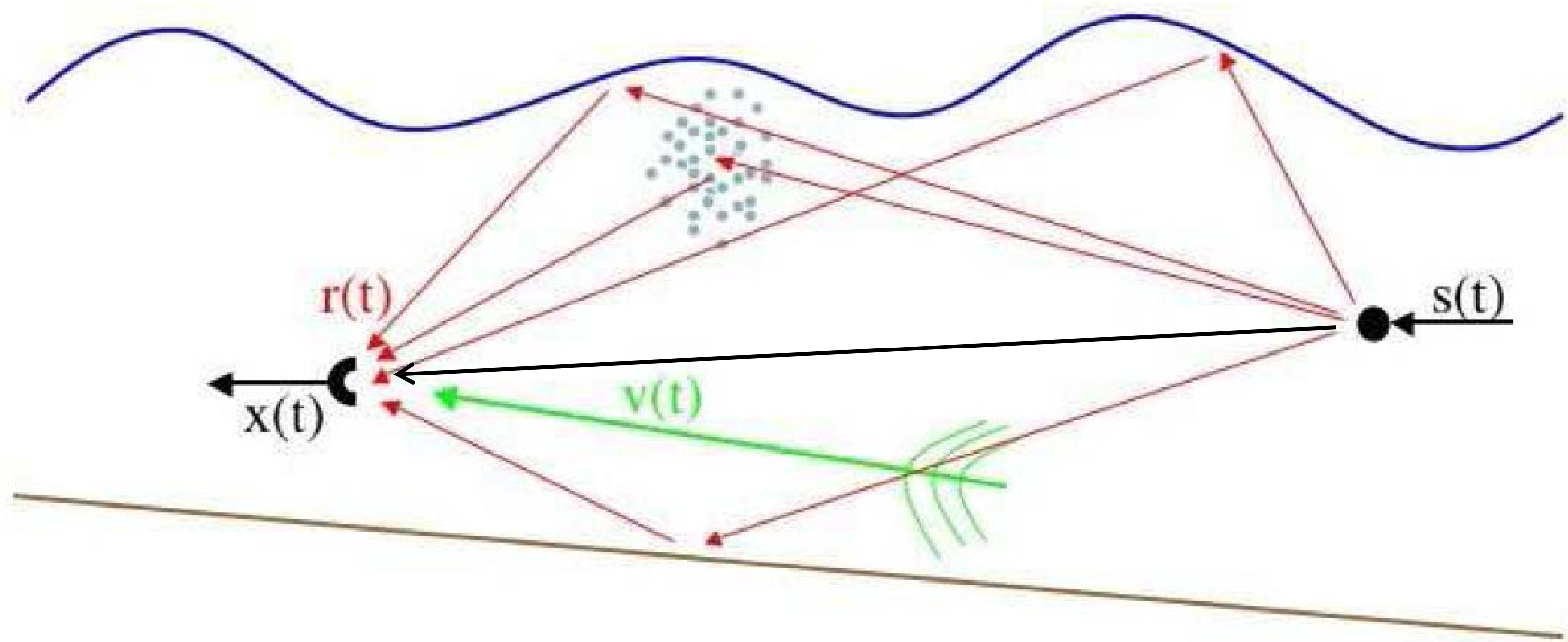


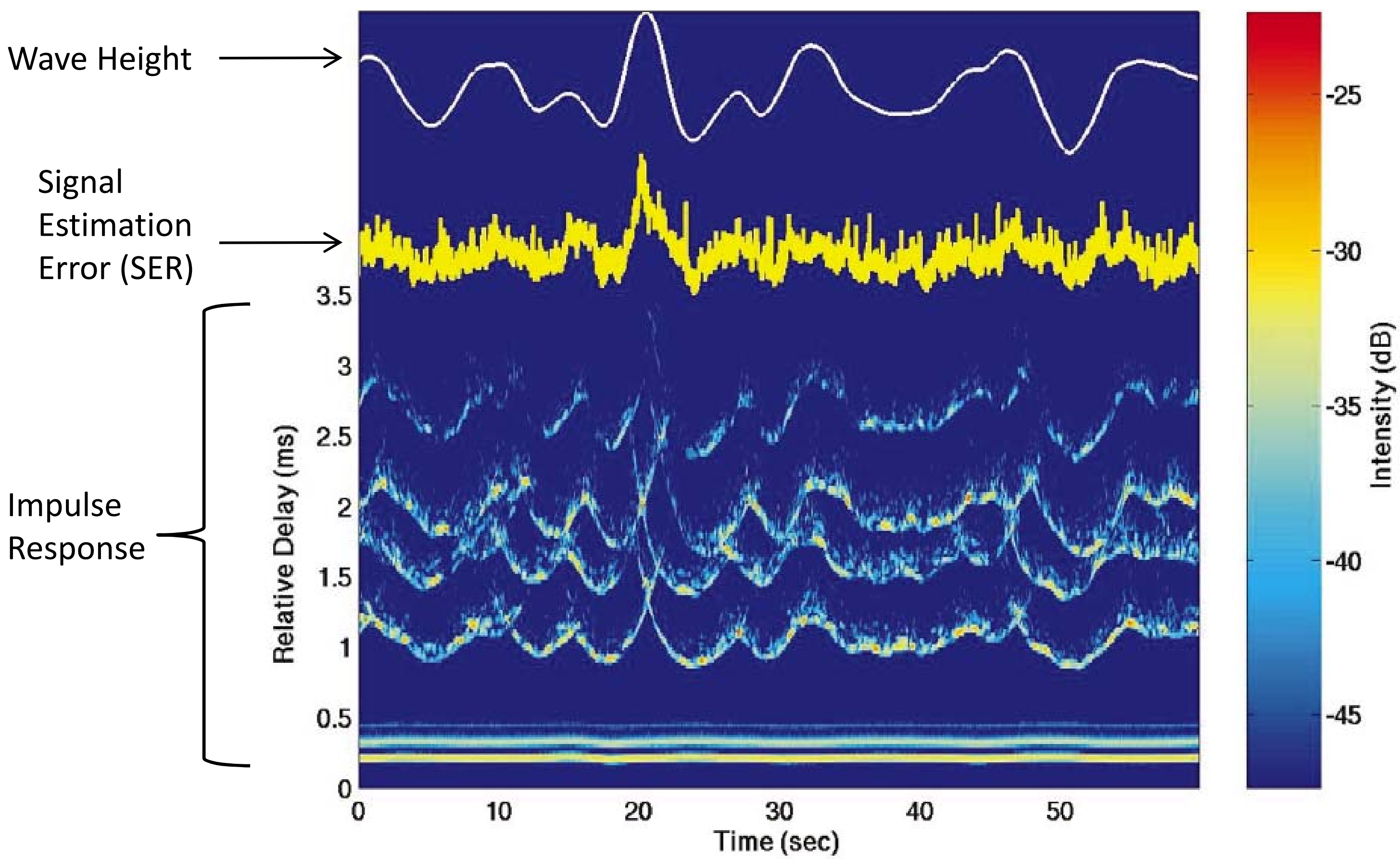
Figure 20: Typical Shallow Water Channel Scattering Function

Preisig, SPACE02

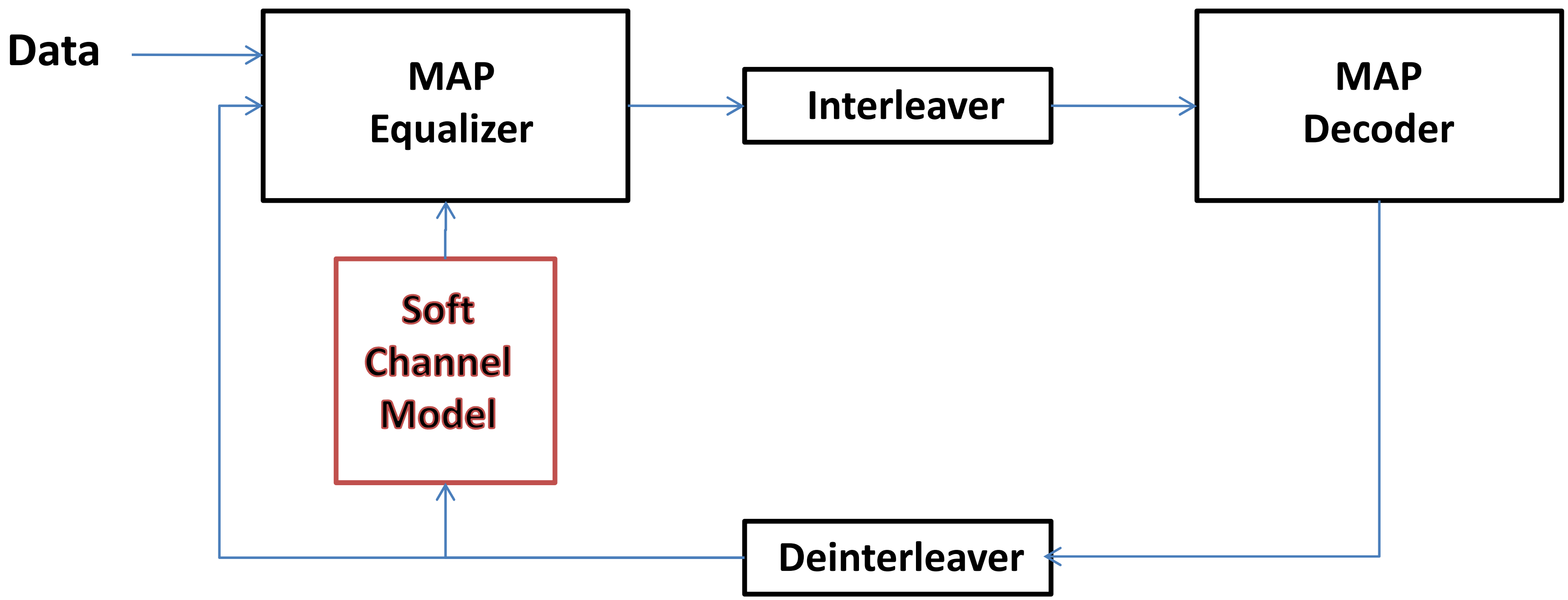
Underwater Signal Paths



Dynamic Channel



Turbo Equalization



Channel Model

- Finite impulse response (FIR) channel

$$\mathbf{y}\ [n] = \mathbf{\Phi}\ \mathbf{g}\ ^*[n] + \mathbf{v}\ [n]$$

$$\mathbf{y}\ [n] = \begin{bmatrix} y[n] \\ y[n-1] \\ \vdots \\ y[n-K+1] \end{bmatrix} \quad \mathbf{g}\ [n] = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{M-1} \end{bmatrix} \quad \mathbf{\Phi}\ = \begin{bmatrix} x[n] & x[n-1] & \cdots & x[n-M+1] \\ x[n-1] & x[n-2] & \cdots & x[n-M] \\ \vdots & \vdots & \ddots & \vdots \\ x[n-K+1] & x[n-K] & \cdots & x[n-K-M+2] \end{bmatrix}$$

Typical channel length, M≈50 for 12kHz carrier, 100m depth, 4kbps data rate

Soft Channel Model (Song2004)

- Data Model: $(x[n] = \bar{x}[n] + \xi[n])$

- Cost Function: $J(\hat{g}) = \sum_{k=0}^{N} \lambda^{n-k} \frac{|y[k] - \bar{x}[k]^T \hat{\mathbf{g}}^*|^2}{q[n]}$

- Solution: $\hat{\mathbf{g}}[n] = \left(\sum_{k=0}^{n} \frac{\lambda^{n-k}}{q[k]} \mathbf{x}[k]\mathbf{x}^T[k] \right)^{-1} \left(\sum_{k=0}^{n} \frac{\lambda^{n-k}}{q[n]} \bar{\mathbf{x}}[k] y^*[k] \right)$

Soft RLS Algorithm

$$\hat{\mathbf{g}}[n] = \mathbf{\Phi}_n^{-1}\mathbf{\Theta}_n$$

$$\mathbf{\Phi}'[n] = \sum_{k=0}^{n} \frac{\lambda^{n-k}}{q[k]}\bar{\mathbf{x}}[k]\bar{\mathbf{x}}^T[k] = \lambda\mathbf{\Phi}'[n-1] + \frac{\bar{\mathbf{x}}[n]\bar{\mathbf{x}}^T[n]}{q[n]}$$

$$\mathbf{\Theta}'[n] = \sum_{k=0}^{n} \frac{\lambda^{n-k}}{q[n]}\bar{\mathbf{x}}[k]y^*[k] = \lambda\mathbf{\Theta}'[n-1] + \frac{\bar{\mathbf{x}}[n]y^*[n]}{q[n]}$$

$$P[n] = \mathbf{\Phi}'^{-1}[n]$$

$$e[n] = y[n] - \hat{\mathbf{g}}[n]\bar{\mathbf{x}}[n]$$

$$\mathbf{k}[n] = \frac{P[n-1]\bar{\mathbf{x}}[n]}{\lambda q[n] + \bar{\mathbf{x}}^H[n]P[n-1]\bar{\mathbf{x}}[n]}$$

$$\hat{\mathbf{g}}[n] = \hat{\mathbf{g}}[n-1] + \mathbf{k}[n]e^*[n]$$

$$P[n] = \frac{1}{\lambda}(I - \mathbf{k}[n]\bar{\mathbf{x}}[n])P[n-1]$$

Matching Pursuit

$$\textit{initialization} \quad (27)$$

$$\mathbf{r}_0 = \mathbf{y} \quad (28)$$

$$b_{0,j} = \mathbf{c}_j^h \mathbf{r}_0, \quad \text{for } j = 1, \cdots, K \quad (29)$$

$$s_1 = \arg \max_{j=1,\cdots,K} \frac{|b_{0,j}|^2}{\|\mathbf{c}_j\|^2} \quad (30)$$

$$I_1 = \{s_1\} \quad (31)$$

$$\widehat{x}_1 = \frac{b_{0,s_1}}{\|\mathbf{c}_{s_1}\|^2} \quad (32)$$

$$b_{1,j} = b_{0,j} - \widehat{x}_1 \mathbf{c}_j^h \mathbf{c}_{s_1}, \quad \text{for } j = 1, \cdots, K,\ j \notin I_1 \quad (33)$$

$$\textit{the pth iteration, } p > 1 \quad (34)$$

$$s_p = \arg \max_{j=1,\cdots,K, j \notin I_{p-1}} \frac{|b_{p-1,j}|^2}{\|\mathbf{c}_j\|^2} \quad (35)$$

$$I_p = \{I_{p-1}\ ,\ s_p\} \quad (36)$$

$$\widehat{x}_p = \frac{b_{p-1,s_p}}{\|\mathbf{c}_{s_p}\|^2} \quad (37)$$

$$b_{p,j} = b_{p-1,j} - \widehat{x}_p \mathbf{c}_j^h \mathbf{c}_{s_p}, \quad \text{for } j = 1, \cdots, K,\ j \notin I_p \quad (38)$$

TABLE I

THE BASIC MATCHING PURSUIT ALGORITHM (MP).

Other variations: Orthogonal MP (OMP), Order Recursive Least Squares MP, etc.

Problems

- RLS does not take advantage of sparsity
- Matching pursuit not adapted for soft data
- Errors in matching pursuit “dictionary” still an open problem

Ideas

- Use two step approach
  - Identify energetic taps (maybe matching pursuit)
  - Use RLS technique with only those taps
- Use weighted matching pursuit
  - Directly adapt matching pursuit to handle soft data

More ideas?

- Compressed sensing?
  - Does not use all of our knowledge about channel, but still might be good?

- Something else?
  - Need technique that uses soft data and sparse channel model

Questions?