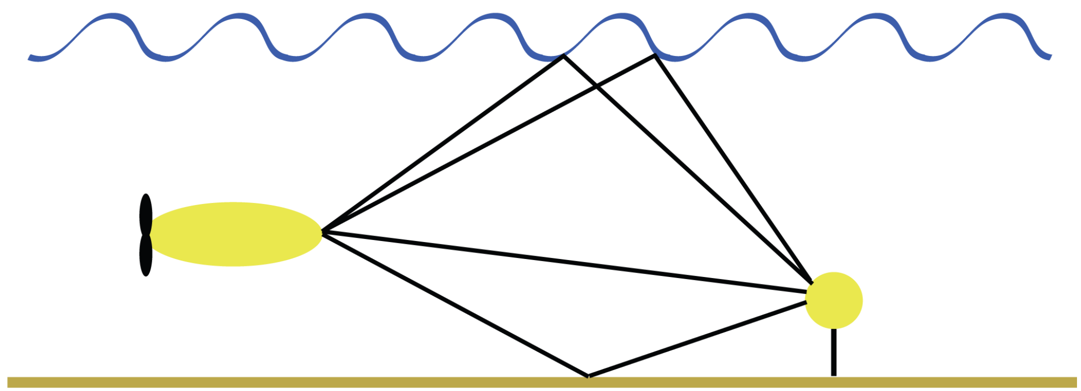
Comparison and Analysis of Equalization Techniques for the Time-Varying Underwater Acoustic Channel



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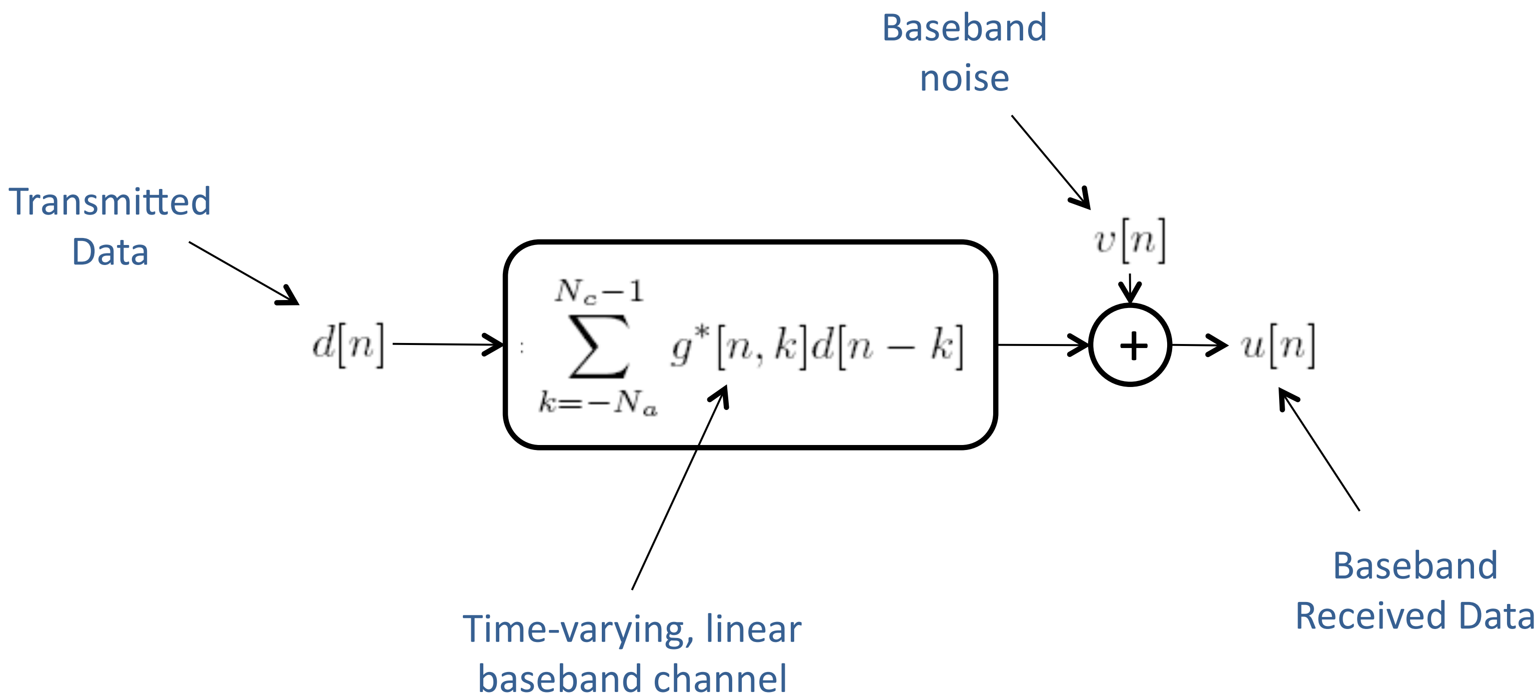
158th Meeting of the Acoustical Society of America

San Antonio

Channel Model



Matrix-vector Form:

$$\mathbf{u}[n] = \mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] = \overbrace{\mathbf{G}_{\mathbf{fb}}[n]\mathbf{d}_{\mathbf{fb}}[n] + \mathbf{G}_{0}[n]\mathbf{d}_{0}[n]}^{\textit{Split Channel Convolution Matrix}} + \mathbf{v}[n]$$

Decision Feedback Equalizer (DFE)

- Problem Setup: $\hat{\mathbf{h}}_{\mathrm{opt}} = \arg\min_{\mathbf{h}'} \mathrm{E}\{|\mathbf{h}^H \mathbf{z} - d|^2\}$
- Estimate using RX data and TX data estimates

$$\mathbf{z}[n] \;=\; [u[n-Lc+1]\ \ldots\ u[n]\ \ldots\ u[n+L_a],\ \hat{d}[n-1]\ \ldots\ \hat{d}[n-L_{fb}]]^T$$

- DFE Eq: $\hat{d}[n] = \hat{\mathbf{h}}_{\mathrm{opt}}^H[n]\mathbf{z}[n] = \mathbf{h}_{\mathbf{ff}}^H \mathbf{u}[\mathbf{n}] + \mathbf{h}_{\mathbf{fb}}^H \hat{\mathbf{d}}_{\mathbf{fb}}$

Solution to Weiner-Hopf Eq.

$$\begin{aligned} \hat{\mathbf{h}}_{\mathrm{opt}}[n] &= \mathbf{R_z}^{-1}[n]\mathbf{r_{zd}}[n] \\ \mathbf{R_z}[n] &= \mathrm{E}\{\mathbf{z}\mathbf{z}^H\} \\ \mathbf{r_{zd}} &= \mathrm{E}\{\mathbf{z}d^*\} \end{aligned}$$

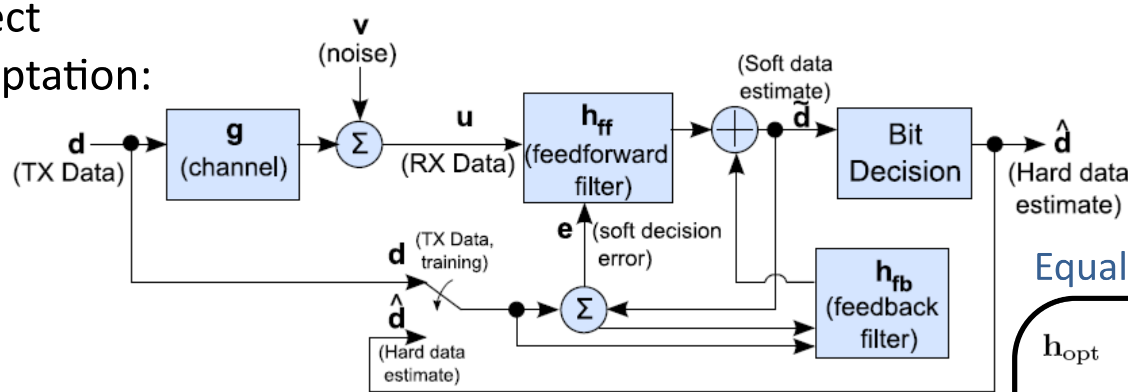
MMSE Sol. Using Channel Model

$$\begin{aligned} \mathbf{h_{ff}} &= [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R_v}]^{-1}\mathbf{g}_0 \\ \mathbf{h_{fb}} &= -\mathbf{G_{fb}}\mathbf{h_{ff}} \end{aligned}$$

- Two Parts:
  - (Linear) feed-forward filter (of RX data)
  - (Linear) feedback filter (of data estimates)

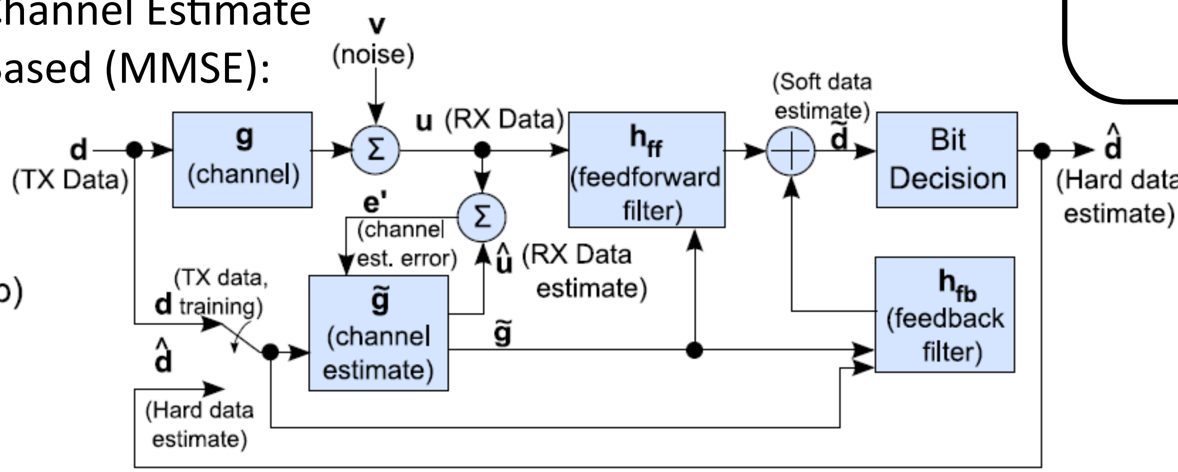
DFE Strategies

Direct Adaptation:



(a)

Channel Estimate Based (MMSE):



(b)

Equalizer Tap Solution:

$$
\begin{aligned}
\mathbf{h}_{opt} &= \begin{bmatrix} \mathbf{h}_{ff} \\ \mathbf{h}_{fb} \end{bmatrix} \\
&= \left(\mathrm{E}[\mathbf{z}\mathbf{z}^H]\right)^{-1} \mathrm{E}[\mathbf{z}d^*] \\
&= \left(\mathrm{E}\left[\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{fb} \end{bmatrix}\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{fb} \end{bmatrix}^H\right]\right)^{-1} \mathrm{E}\left[\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{fb} \end{bmatrix} d^*\right] \\
&= \begin{bmatrix} [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R}_v]^{-1}\mathbf{g}_0 \\ -\mathbf{G}_{fb}\mathbf{h}_{ff} \end{bmatrix}
\end{aligned}
$$

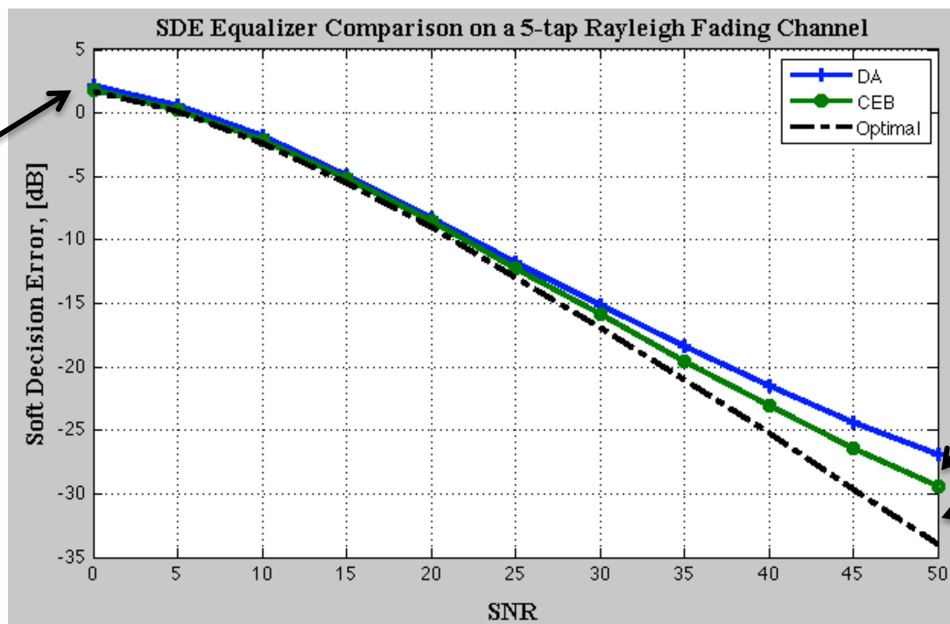
Central Question:

Why is the performance of a channel estimate based equalizer different than a direct adaptation equalizer?

Comparison between DA and CEB

- In the past, CEB methods empirically shown to have lower mean squared error at high SNR
- Reasons for difference varied:
  - Condition number of correlation matrix
  - Num. of samples required to get good estimate

Similar performance at low SNR



3dB performance difference between CEB and DA at high SNR

Performance gap due to channel estimation

Comparison between DA and CEB

- Our analysis shows the answer is:
  Longer *corr. time for channel coefficients than MMSE equalizer coefficients at high SNR*

- Will examine low SNR and high SNR regimes
  - Use simulation to show transition of correlation time for the equalizer coefficients from low to high is smooth

Low SNR Regime

Update eqn. for feed-forward equalizer coefficients (AR model assumed):

$$\mathbf{h_{ff}}[n+1] = (\mathbf{G}_0[n+1]\mathbf{G}_0^H[n+1] + \mathbf{R_v})^{-1}(\mathbf{g}_0[n+1])$$
$$\approx \mathbf{R_v}^{-1}(\alpha \mathbf{g}_0[n] + \mathbf{w}[n])$$
$$\approx \alpha \mathbf{h_{ff}}[n] + \mathbf{R_v}^{-1}\mathbf{w}[n]$$

Approximation:

$$\mathbf{R_v} + \mathbf{G}[n]\mathbf{G}^H[n] \approx \mathbf{R_v}$$

Has same correlation structure as channel coefficients

High SNR

$$\mathbf{h}_{\mathbf{ff}}[n+1] \;=\; (\mathbf{G}_0[n+1]\mathbf{G}_0^H[n+1]+\mathbf{R}_{\mathbf{v}})^{-1}(\mathbf{g}_0[n+1])$$

Approximation:

$$\mathbf{G}_0[n]\mathbf{G}_0^H[n]+\mathbf{R}_{\mathbf{v}} \approx \mathbf{G}_0[n]\mathbf{G}_0^H[n]$$

$$(\mathbf{G}_0[n]\mathbf{G}_0^H[n])\mathbf{h}_{\mathbf{ff}}[n] = \mathbf{g}_0[n]$$

Reduced Channel Convolution Matrix:

$$\mathbf{G}_0 = \begin{bmatrix} g_0^*[n-L+1] & 0 & 0 & \cdots & 0 \\ g_1^*[n-L+2] & g_0^*[n-L+2] & 0 & \cdots & 0 \\ g_2^*[n-L+3] & g_1^*[n-L+3] & g_0^*[n-L+3] & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ g_L^*[n] & g_{L-1}^*[n] & g_{L-2}^*[n] & \cdots & g_0^*[n] \end{bmatrix}$$

Matrix Product:

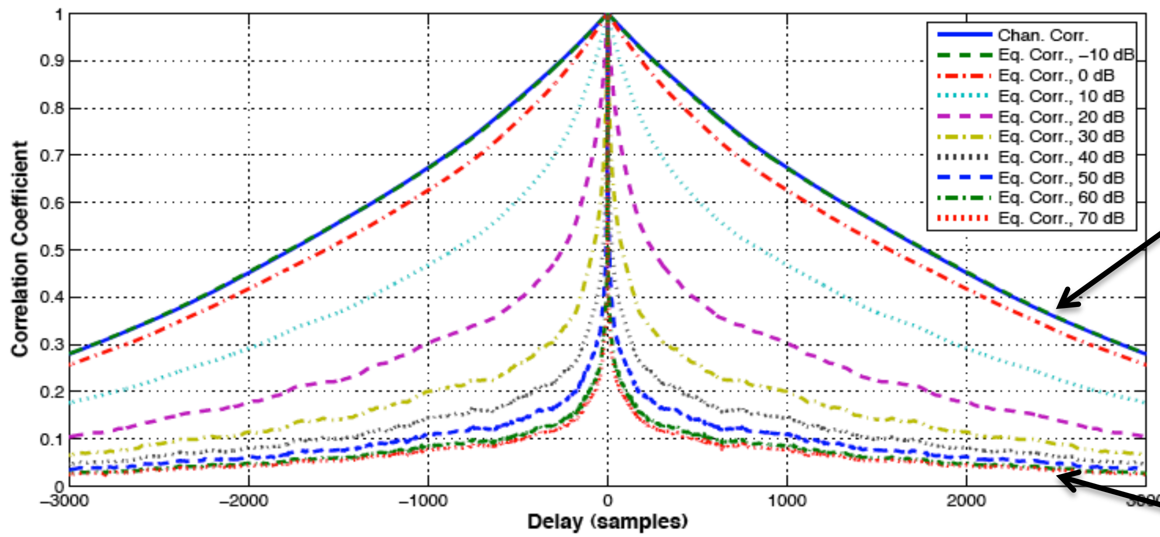
$$\mathbf{G}_0\mathbf{G}_0^H = \begin{bmatrix} |g_0|^2 & \cdots \\ g_0 g_1^* & \cdots \\ g_0 g_2^* & \cdots \\ \vdots & \vdots \\ g_0 g_L & \cdots \end{bmatrix}$$

Reduces to single tap:

$$\mathbf{h}[n] = \begin{bmatrix} 1/g_0 & 0 & 0 & \ldots & 0 \end{bmatrix}^T$$

Correlation over SNR – 1-tap

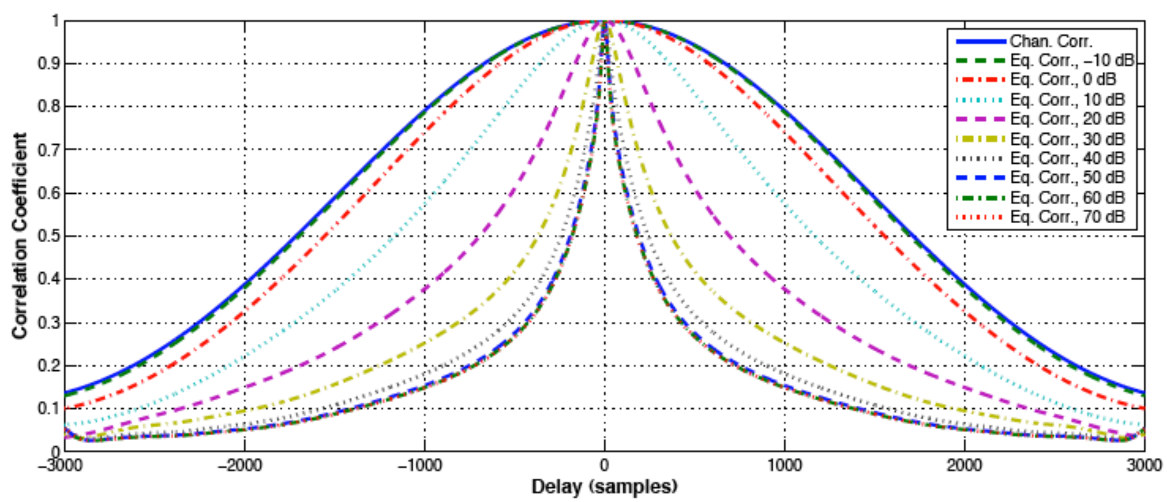
AR(1) model



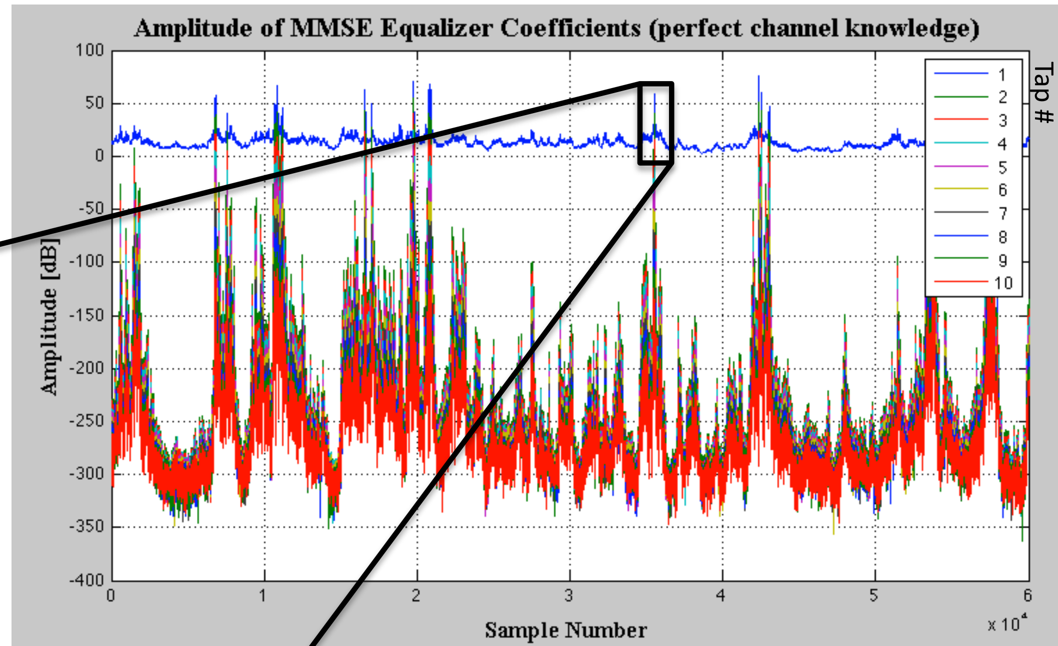
Channel and Equalizer Coeff. Correlation the Same at low SNR

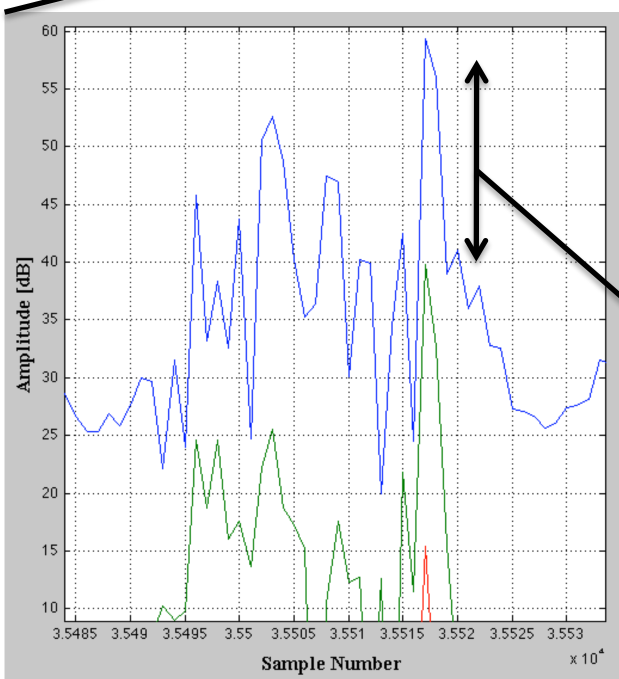
Equalizer Coeff. Correlation reduces as SNR increases

Gaussian model



Amplitude of MMSE Eq. Coeff.

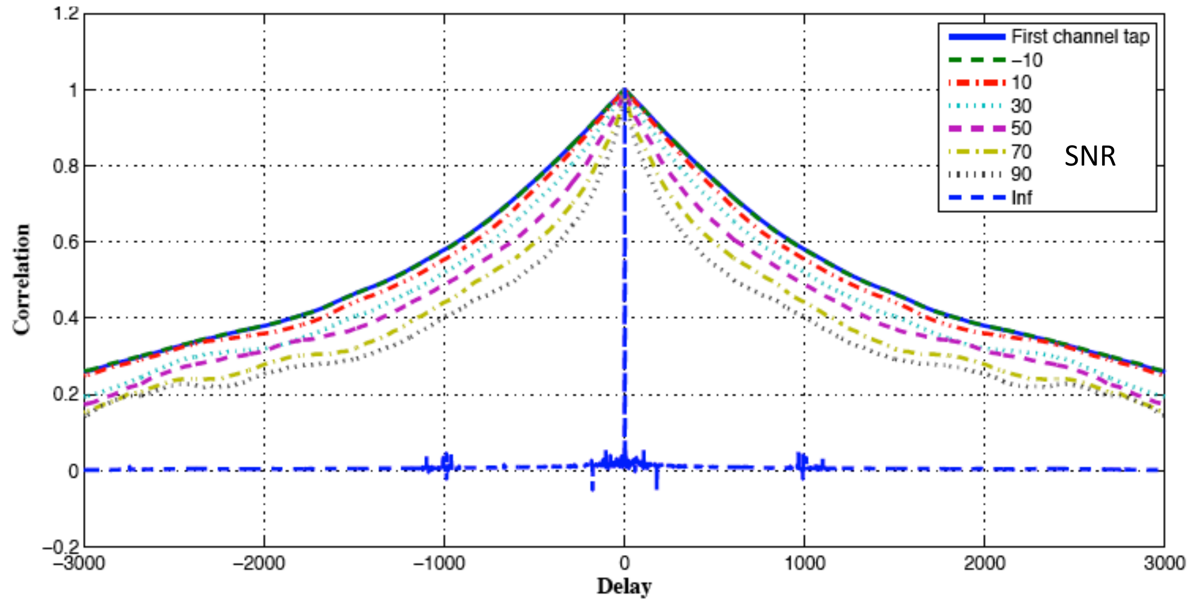


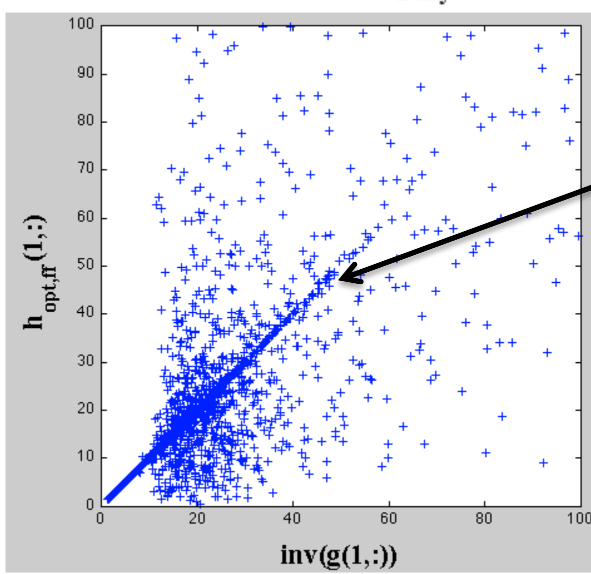


First feed-forward equalizer coefficient has much larger amplitude than others

Multi-tap correlation

Multi-tap AR(1) model





Strong linear correlation between inverse of first channel tap and first MMSE Eq. tap

Take-home Message

- Channel impulse-response taps have longer correlation time than MMSE equalizer taps
  – DA has greater MSE than CEB

- For time-invariant statistics, CEB and DA algorithms have similar performance
  – Low-SNR regime (assuming stationary noise)
  – Underwater channel operates in low SNR regime (<35dB)

Questions?

Backup Slides

Assumptions

- Unit variance, white transmit data

$$\mathrm{E}\{\mathbf{d}[n]\mathbf{d}^H[n]\} = \mathbf{I}$$

- TX data and obs. noise are uncorrelated

$$\mathrm{E}\{\mathbf{v}[n]\mathbf{d}^H[m]\} = \mathbf{0}$$

  - Obs. Noise variance:

$$\mathbf{R}_{\mathbf{v}} = \mathrm{E}\{\mathbf{v}[n]\mathbf{v}^H[n]\}$$

- Perfect data estimation (for feedback)

$$\hat{\mathbf{d}} = \mathbf{d}$$

- Equalizer Length = Estimated Channel Length

$$N_a + N_c = L_a + L_c$$

- MMSE Equalizer Coefficients have form:

$$\begin{aligned} \mathbf{h}_{\mathbf{ff}} &= [\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R}_{\mathbf{v}}]^{-1}\mathbf{G}\mathbf{s} \\ \mathbf{h}_{\mathbf{fb}} &= -\mathbf{G}_{fb}^H\mathbf{h}_{\mathbf{ff}} \end{aligned}$$

WSSUS AR channel model

- Simple channel model to analyze
- Similar to encountered situations

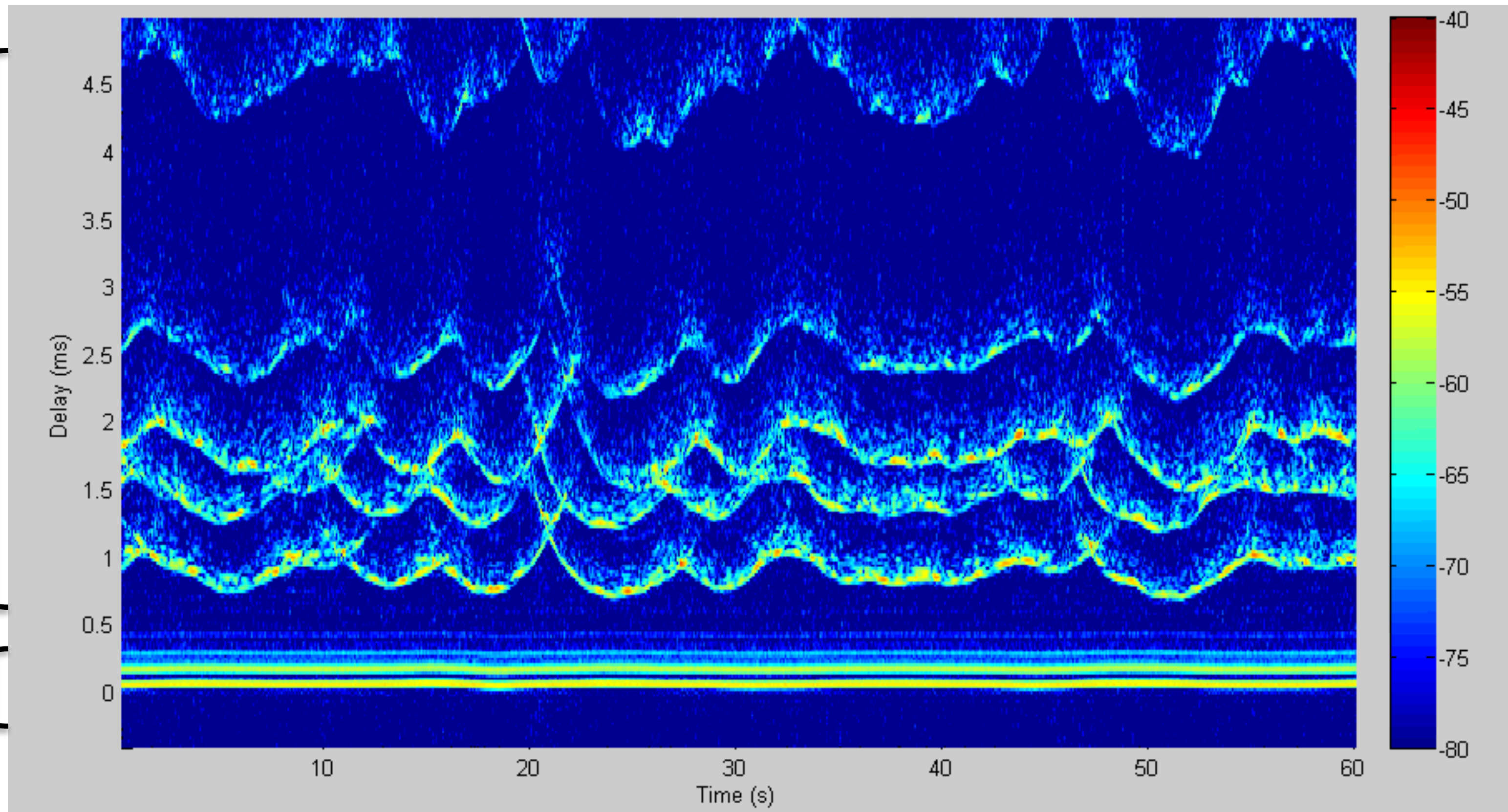
$$g[n+1] = \alpha g[n] + w[n]$$

$$R_{gg}[k] = \mathrm{E}\{g[n]g^*[n+k]\} = \begin{cases} \sigma_w^2 \left(\frac{(\alpha^*)^k}{1-|\alpha|^2}\right) & k \geq 0 \\ \sigma_w^2 \left(\frac{\alpha^{-k}}{1-|\alpha|^2}\right) & k < 0 \end{cases}$$

Time Varying Impulse Response

Wave interacting paths, highly

Direct Path and Bottom Bounce, Time Invariant



Wavefronts II Experiment from San Diego, CA

Future Work

- Reduce time needed to update channel model
  - Sparsity
  - Physical Constraints
  - Optimization Techniques

- Combine DA and CEB equalizers
  - Better performance at lower complexity